Design Considerations for Steel Fiber Reinforced Concrete

Reported by ACI Committee 544

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The present state of development of design practices for fiber reinforced concrete and mortar using steel fibers is reviewed. Mechanical properties are discussed, design methods are presented, and typical applications are listed.

Keywords: beams (supports;) cavitation; compressive strength; concrete slabs; creep properties; fatigue (materials); **fiber reinforced concretes;** fibers; flexural strength; freeze-thaw durability; **metal fibers; mortars (material); structural design.**

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CHAPTER 1-INTRODUCTION

Steel fiber reinforced concrete (SFRC) and mortar made with hydraulic cements and containing fine or fine and coarse aggregates along with discontinuous discrete steel fibers are considered in this report. These materials are routinely used in only a few types of ap-

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plications at present (1988), but ACI Committee 544 believes that many other applications will be developed once engineers become aware of the beneficial properties of the material and have access to appropriate design procedures. The contents of this report reflect the experience of the committee with design procedures now in use.

The concrete used in the mixture is of a usual type, although the proportions should be varied to obtain good workability and take full advantage of the fibers. This may require limiting the aggregate size, optimizing the gradation, increasing the cement content, and perhaps adding fly ash or other admixtures to improve workability. The fibers may take many shapes. Their cross sections include circular, rectangular, half-round, and irregular or varying cross sections. They may be straight or bent, and come in various lengths. A convenient numerical parameter called the aspect ratio is used to describe the geometry. This ratio is the fiber length divided by the diameter. If the cross section is not round, then the diameter of a circular section with the same area is used.

The designer may best view fiber reinforced concrete as a concrete with increased strain capacity, impact resistance, energy absorption, and tensile strength. However, the increase in these properties will vary from substantial to nil depending on the quantity and type of fibers used; in addition, the properties will not increase at the same rate as fibers are added.

Several approaches to designing members with steel fiber reinforced concrete (SFRC) are available that are based on conventional design methods supplemented by special procedures for the fiber contribution. These methods generally modify the internal forces in the member to account for the additional tension from the fibers. When supported by full-scale test data, these approaches can provide satisfactory designs. The major differences in the proposed methods are in the determination of the magnitude of the tensile stress increase due to the fibers and in the manner in which the total force is calculated. Other approaches that have been used are often empirical, and they may apply only in certain cases where limited supporting test data have been obtained. They should be used with caution in new applications, only after adequate investigation.

Generally, for structural applications, steel fibers should be used in a role supplementary to reinforcing bars. Steel fibers can reliably inhibit cracking and improve resistance to material deterioration as a result of fatigue, impact, and shrinkage, or thermal stresses. A conservative but justifiable approach in structural members where flexural or tensile loads occur, such as in beams, columns, or elevated slabs (i.e., roofs, floors, or slabs not on grade), is that reinforcing bars must be used to support the total tensile load. This is because the variability of fiber distribution may be such that low fiber content in critical areas could lead to unacceptable reduction in strength.

In applications where the presence of continuous reinforcement is not essential to the safety and integrity of the structure, e.g., floors on grade, pavements, overlays, and shotcrete linings, the improvements in flexural strength, impact resistance, and fatigue performance associated with the fibers can be used to reduce section thickness, improve performance, or both.

ACI 318 does not provide for use of the additional tensile strength of the concrete in building design and, therefore, the design of reinforcement must follow the usual procedure. Other applications provide more freedom to take full advantage of the improved properties of SFRC.

There are some applications where steel fibers have been used without bars to carry flexural loads. These have been short-span elevated slabs, e.g., a parking garage at Heathrow Airport with slabs 3 ft-6 in. (1.07 m) square by 2½ in. (10 cm) thick, supported on four sides (Anonymous 1971). In such cases, the reliability of the members should be demonstrated by full-scale load tests, and the fabrication should employ rigid quality control.

Some full-scale tests have shown that steel fibers are effective in supplementing or replacing the stirrups in beams (Williamson 1978; Craig 1983; Sharma 1986). Although it is not an accepted practice at present, other full-scale tests have shown that steel fibers in combination with reinforcing bars can increase the moment capacity of reinforced concrete beams (Henager and Doherty 1976; Henager 1977a).

Steel fibers can also provide an adequate internal restraining mechanism when shrinkage-compensating cements are used, so that the concrete system will perform its crack control function even when restraint from conventional reinforcement is not provided. Fibers and shrinkage-compensating cements are not only compatible, but complement each other when used in combination (Paul et al. 1981). Guidance concerning shrinkage-compensating cement is available in ACI 223.1R.

ASTM A 820 covers steel fibers for use in fiber reinforced concrete. The design procedures discussed in this report are based on fibers meeting that specification.

Additional sources of information on design are available in a selected bibliography prepared by Hoff (1976-1982), in ACI publications SP-44 (1974) and SP-81 (1984), in proceedings of the 1985 U.S.-Sweden joint seminar edited by Shah and Skarendahl (1986), and the recent ACI publication SP-105 edited by Shah and Batson (1987).

For guidance regarding proportioning, mixing, placing, finishing, and testing for workability of steel fiber reinforced concrete, the designer should refer to ACI 544.3R.

CHAPTER 2-MECHANICAL PROPERTIES USED IN DESIGN

2.1-General

The mechanical properties of steel fiber reinforced concrete are influenced by the type of fiber; length-to-diameter ratio (aspect ratio); the amount of fiber; the strength of the matrix; the size, shape, and method of preparation of the specimen; and the size of the aggregate. For this reason, mixtures proposed for use in design should be tested, preferably in specimens representing the end use, to verify the property values assumed for design.

SFRC mixtures that can be mixed and placed with conventional equipment and procedures use from 0.5 to 1.5 volume percent* fibers. However, higher percentages of fibers (from 2 to 10 volume percent) have been used with special fiber addition techniques and placement procedures (Lankard 1984). Most properties given in this chapter are for the lower fiber percentage range. Some properties, however, are given for the higher fiber percentage mixtures for information in applications where the additional strength or toughness may justify the special techniques required.

Fibers influence the mechanical properties of concrete and mortar in all failure modes (Gopalaratnam and Shah 1987a), especially those that induce fatigue and tensile stress, e.g., direct tension, bending, impact, and shear. The strengthening mechanism of the fibers involves transfer of stress from the matrix to the fiber by interfacial shear, or by interlock between the fiber and matrix if the fiber surface is deformed. Stress is thus shared by the fiber and matrix in tension until the matrix cracks, and then the total stress is progressively transferred to the fibers.

Aside from the matrix itself, the most important variables governing the properties of steel fiber reinforced concrete are the fiber efficiency and the fiber content (percentage of fiber by volume or weight and total number of fibers). Fiber efficiency is controlled by the resistance of the fibers to pullout, which in turn depends on the bond strength at the fiber-matrix interface. For fibers with uniform section, pullout resistance increases with an increase in fiber length; the longer the fiber the greater its effect in improving the properties of the composite.

Also, since pullout resistance is proportional to interfacial surface area, nonround fiber cross sections and smaller diameter round fibers offer more pullout resistance per unit volume than larger diameter round fibers because they have more surface area per unit volume. Thus, the greater the interfacial surface area (or the smaller the diameter), the more effectively the fibers bond. Therefore, for a given fiber length, a high ratio of length to diameter (aspect ratio) is associated with high fiber efficiency. On this basis, it would appear that the fibers should have an aspect ratio high enough to insure that their tensile strength is approached as the composite fails.

Unfortunately, this is not practical. Many investigations have shown that use of fibers with an aspect ratio greater than 100 usually causes inadequate workability of the concrete mixture, non-uniform fiber distribution, or both if the conventional mixing techniques are used (Lankard 1972). Most mixtures used in practice employ fibers with an aspect ratio less than 100, and failure of the composite, therefore, is due primarily to fiber pullout. However, increased resistance to pullout without increasing the aspect ratio is achieved in fibers with deformed surfaces or end anchorage; failure may involve fracture of some of the fibers, but it is still usually governed by pullout.

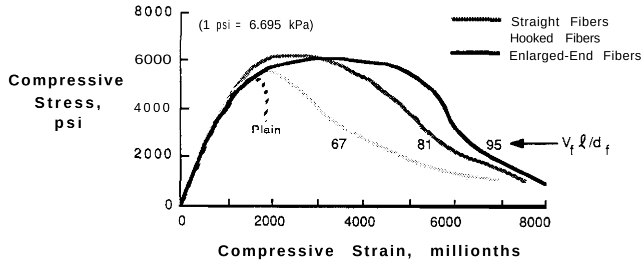

Fig. 2.1-Stress-strain curves for steel fiber reinforced concrete in compression, ⅜ -in. (9.5-mm) aggregate mixtures (Shah 1978)

An advantage of the pullout type of failure is that it is gradual and ductile compared with the more rapid and possibly catastrophic failure that may occur if the fibers break in tension. Generally, the more ductile the steel fibers, the more ductile and gradual the failure of the concrete. Shah and Rangan (1970) have shown that the ductility provided by steel fibers in flexure was enhanced when the high-strength fibers were annealed (a heating process that softens the metal, making it less brittle).

An understanding of the mechanical properties of SFRC and their variation with fiber type and amount is an important aspect of successful design. These properties are discussed in the remaining sections of this chapter.

2.2-Compression

The effect of steel fibers on the compressive strength of concrete is variable. Documented increases for concrete (as opposed to mortar) range from negligible in most cases to 23 percent for concrete containing 2 percent by volume of fiber with ℓ/d = 100, ¾-in. (19-mm) maximum-size aggregate, and tested with 6 x 12 in. (150 x 300 mm) cylinders (Williamson 1974). For mortar mixtures, the reported increase in compressive strength ranges from negligible (Williamson 1974) to slight (Fanella and Naaman 1985).

Typical stress-strain curves for steel fiber reinforced concrete in compression are shown in Fig. 2.1 (Shah et al. 1978). Curves for steel fiber reinforced mortar are shown in Fig. 2.2 and 2.3 (Fanella and Naaman 1985). In these curves, a substantial increase in the strain at the peak stress can be noted, and the slope of the descending portion is less steep than that of control specimens without fibers. This is indicative of substantially higher toughness, where toughness is a measure of ability to absorb energy during deformation, and it can be estimated from the area under the stress-strain curves or load-deformation curves. The improved toughness in compression imparted by fibers is useful in

* Percent by volume of the total concrete mixture.

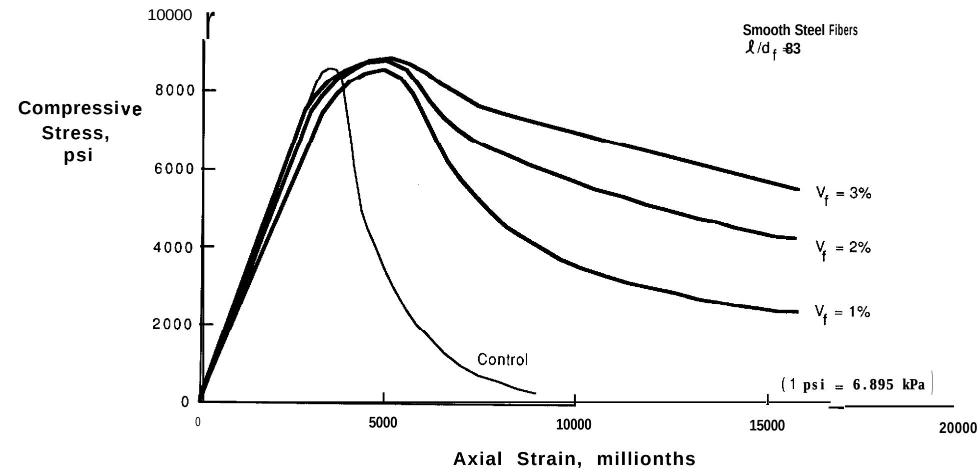

Fig. 2.2-Influence of the volume fraction of fibers on the compressive stress-strain curve

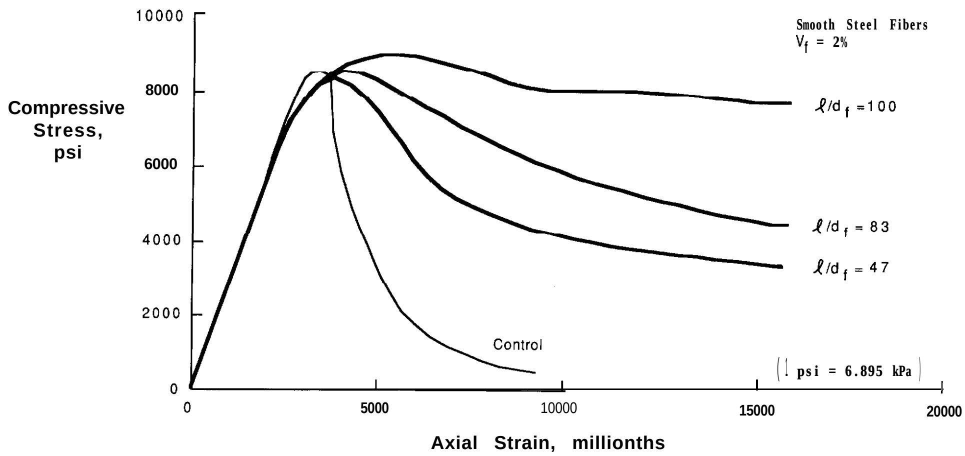

Fig. 2.3-Influence of the aspect ratio of fibers on the stress-strain curve

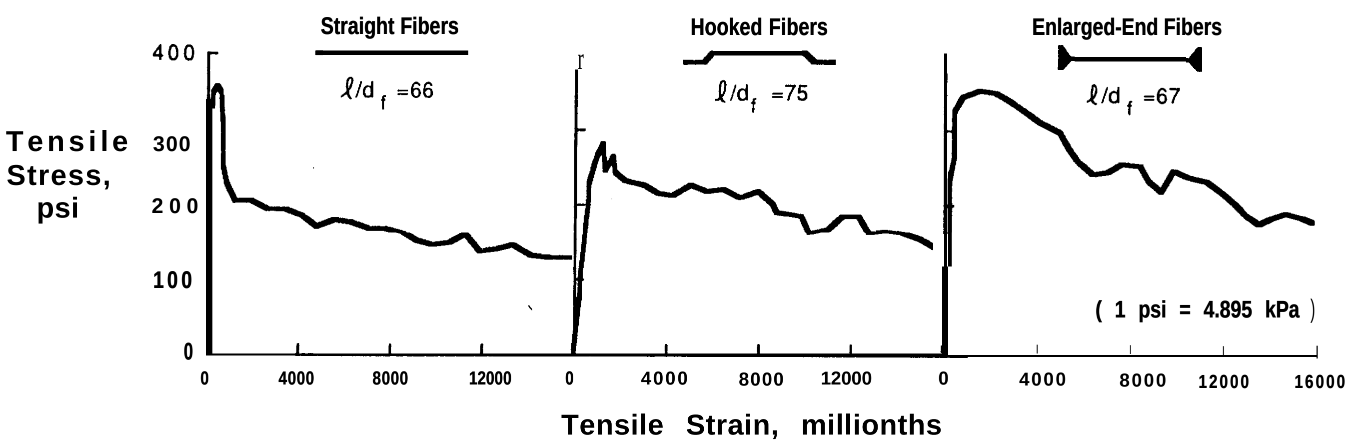

Fig. 2.4-Stress-strain curves for steel fiber reinforced mortars in tension (1.73 percent fibers by volume) (Shah 1978)

preventing sudden and explosive failure under static loading, and in absorbing energy under dynamic loading.

2.3-Direct tension

No standard test exists to determine the stress-strain curve of fiber reinforced concrete in direct tension. The observed curve depends on the size of the specimen, method of testing, stiffness of the testing machine, gage length, and whether single or multiple cracking occurs within the gage length used. Typical examples of stress-strain curves (with strains measured from strain gages) for steel fiber reinforced mortar are shown in Fig. 2.4 (Shah et al. 1978). The ascending part of the curve up to first cracking is similar to that of unreinforced mortar. The descending part depends on the fiber reinforcing parameters, notably fiber shape, fiber amount and aspect ratio.

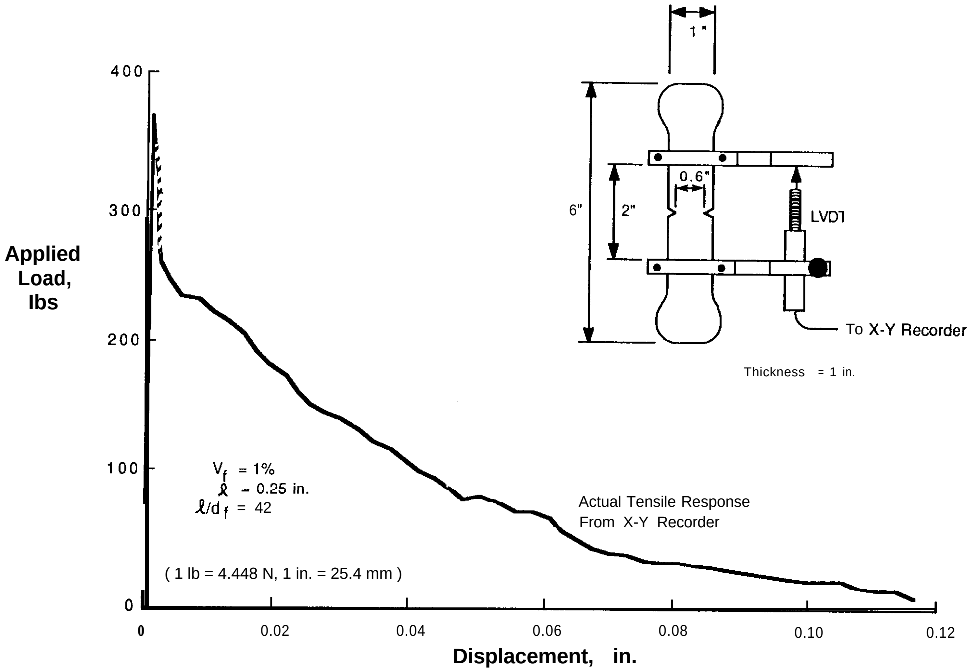

Fig. 2.5-Typical tensile load-versus-displacement curve of steel fiber reinforced mortar (Visalvanich and Naaman 1983)

An investigation of the descending, or post-cracking, portion of the stress-strain curve has led to the data shown in Fig. 2.5 and 2.6 and the prediction equation shown in Fig. 2.6 (Visalvanich and Naaman 1983). If only one crack forms in the tension specimen, as in the tests in Fig. 2.5, deformation is concentrated at the crack, and calculated strain depends on the gage length. Thus, post-crack strain information must be interpreted with care in the post-crack region (Gopalaratnam and Shah 1987b).

The strength of steel fiber reinforced concrete in direct tension is generally of the same order as that of unreinforced concrete, i.e., 300 to 600 psi (2 to 4 MPa). However, its toughness (as defined and measured according to ASTM C 1018) can be one to two orders of magnitude higher, primarily because of the large frictional and fiber bending energy developed during fiber pullout on either side of a crack, and because of deformation at multiple cracks when they occur (Shah et al. 1978; Visalvanich and Naaman 1983; Gopalaratnam and Shah 1987b).

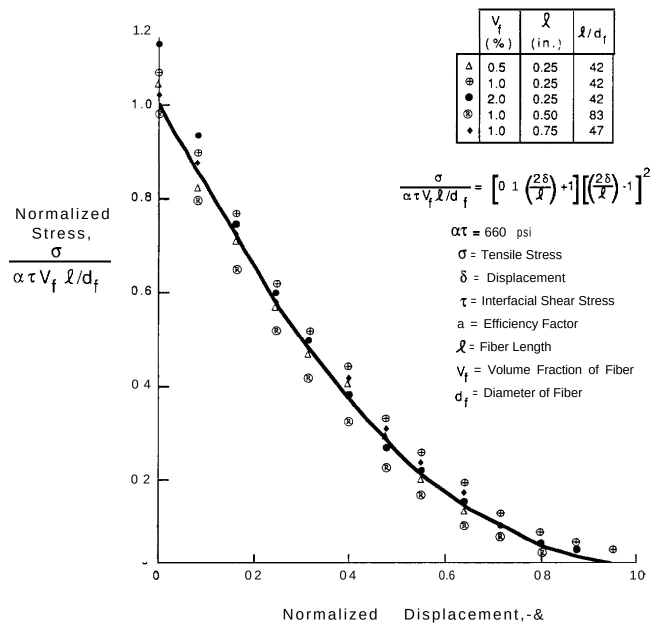

Fig. 2.6-Normalized stress-displacement law of steel fiber reinforced mortar (all cases) (Visalvanich and Naaman 1983)

2.4- Flexural strength

The influence of steel fibers on flexural strength of concrete and mortar is much greater than for direct tension and compression. Two flexural strength values are commonly reported. One, termed the first-crack flexural strength, corresponds to the load at which the load-deformation curve departs from linearity (Point A on Fig. 2.7). The other corresponds to the maximum load achieved, commonly called the ultimate flexural strength or modulus of rupture (Point C on Fig. 2.7). Strengths are calculated from the corresponding load using the formula for modulus of rupture given in ASTM C 78, although the linear stress and strain distributions on which the formula is based no longer apply after the matrix has cracked.

Fig. 2.8 shows the range of flexural load-deflection curves that can result when different amounts and types of fibers are used in a similar matrix and emphasizes the confusion that can occur in reporting of first-crack and ultimate flexural strength. For larger amounts of fibers the two loads are quite distinct (upper curve), but for smaller fiber volumes the first-crack load may be the maximum load as well (lower curves). The shape of

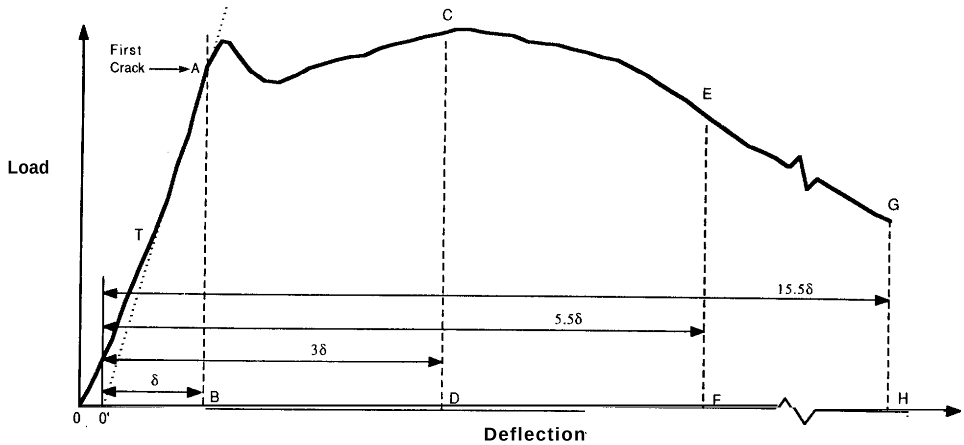

Fig. 2.7-Important characteristics of the load-deflection curve (ASTM C 1018)

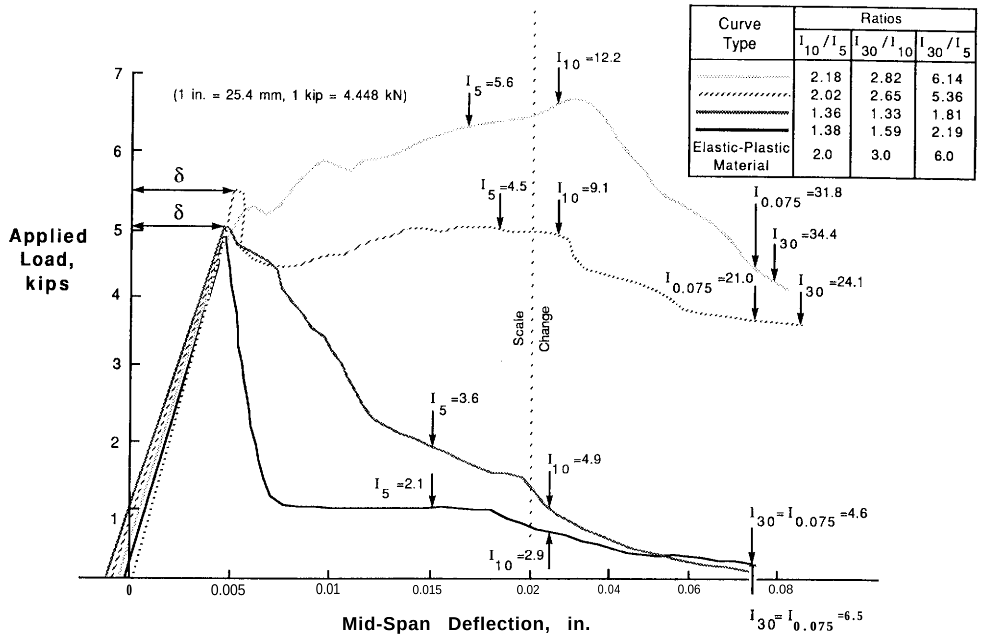

Fig. 2.8-Load-deflection curves illustrating the range of material behavior possible for four mixtures containing various amounts and types of fibers (Johnston 1982b)

the post-cracking curve is an important consideration in design, and this will be discussed relative to the calculation of flexural toughness. It is important, however, that the assumptions on which strength calculations are based be clearly indicated.

Procedures for determining first-crack and ultimate flexural strengths, as published in ACI 544.2R and ASTM C 1018, are based on testing 4 x 4 x 14 in. (100 x 100 x 350 mm) beams under third-point loading for quality control. Other sizes and shapes give higher or lower strengths, depending on span length, width and depth of cross section, and the ratio of fiber length to the minimum cross-sectional dimension of the test specimen.

It is possible, however, to correlate the results obtained in different testing configurations to values for standard beams tested under third-point loading, even when centerpoint loading is employed (Johnston 1982a). This is necessary when attempting to relate the performance of a particular design depth or thickness of material, e.g., a sample obtained from a pavement overlay or shotcrete lining, to the performance of standard 4 x 4 x 14 in. (100 x 100 x 350 mm) beams. The requirements relating cross-sectional size to design thickness of fiber reinforced concrete and to fiber length in ASTM C 1018 state that, for normal thickness of sections or mass concrete applications, the minimum cross-sectional dimension shall be at least three times the fiber length and the nominal maximum aggregate size.

Ultimate flexural strength generally increases in relation to the product of fiber volume concentration v and aspect ratio ℓ/d. Concentrations less than 0.5 volume percent of low aspect ratio fibers (say less than 50) have negligible effect on static strength properties. Prismatic fibers, or hooked or enlarged end (better anchorage) fibers, have produced flexural strength increases over unreinforced matrices of as much as 100 percent

(Johnston 1980). Post-cracking load-deformation characteristics depend greatly on the choice of fiber type and the volume percentage of the specific fiber type used. The cost effectiveness of a particular fiber type/amount combination should therefore be evaluated by analysis or prototype testing.

High flexural strengths are most easily achieved in mortars. Typical values for mortars (w/c ratio = 0.45 to 0.55) are in the range of 1000 to 1500 psi (6.5 to 10 MPa) for 1.5 percent by volume of fibers depending on the ℓ/d and the type of fiber, and may approach 1900 psi (13 MPa) for 2.5 percent by volume of fibers (Johnston 1980).

For fiber reinforced concretes, strengths decrease with increases in the maximum size and proportion of coarse aggregate present. In the field, workability considerations associated with conventional placement equipment and practices usually limit the product of fiber concentration by volume percent and fiber aspect ratio $v\ell/d$ to about 100 for uniform straight fibers. Twenty-eight day ultimate flexural strengths for concretes containing 0.5 to 1.5 percent by volume of fibers with ¼ to ¾ in. (8 to 19 mm) aggregate are typically in the range of 800 to 1100 psi (5.5 to 7.5 MPa) depending on $v\ell/d$, fiber type, and water-cement ratio.

Crimped fibers, surface-deformed fibers, and fibers with end anchorage produce strengths above those for smooth fibers of the same volume concentration, or allow similar strengths to be achieved with lower fiber concentrations. The use of a superplasticizing admixture may increase strengths over the value obtained without the admixture if the w/c ratio is reduced (Ramakrishnan and Coyle 1983).

2.5- Flexural toughness

Toughness is an important characteristic for which steel fiber reinforced concrete is noted. Under static loading, flexural toughness may be defined as the area under the load-deflection curve in flexure, which is the total energy absorbed prior to complete separation of the specimen (ACI 544.1R). Typical load-deflection curves for concrete with different types and amounts of fiber are shown in Fig. 2.8 (Johnston 1982b). Flexural toughness indexes may be calculated as the ratio of the area under the load-deflection curve for the steel fiber concrete to a specified endpoint, to the area up to first crack, as shown in ASTM C 1018, or to the area obtained for the matrix without fibers.

Some examples of index values computed using a fixed deflection of 0.075 in. (1.9 mm) to define the test endpoint for a 4 x 4 x 14 in. (100 x 100 x 350 mm) beam are shown in Fig. 2.8. Examples of index values I_5, I_{10}, and I_{30}, which can be computed for any size or shape of specimen, are also shown in Fig. 2.8.

These indexes, defined in ASTM C 1018, are obtained by dividing the area under the load-deflection curve, determined at a deflection that is a multiple of the first-crack deflection, by the area under the curve up to the first crack. I_5 is determined at a deflection 3 times the first-crack deflection, I_{10} is determined at 5.5, and I_{30} at 15.5 times the first-crack deflection. For example, for the second highest curve of Fig. 2.8, the first-crack deflection is 0.0055 in, (0.014 mm). I_5 is therefore determined at a deflection of 0.0165 in. (0.042 mm). The other values are computed similarly. ASTM C 1018 recommends that the end-point deflection and the corresponding index be selected to reflect the level of serviceability required in terms of cracking and deflection.

Values of the ASTM C 1018 toughness indexes depend primarily on the type, concentration, and aspect ratio of the fibers, and are essentially independent of whether the matrix is mortar or concrete (Johnston and Gray 1986). Thus, the indexes reflect the toughening effect of the fibers as distinct from any strengthening effect that may occur, such as an increase in first-crack strength.

Strengthening effects of this nature depend primarily on matrix characteristics such as water-cement ratio. In general, crimped fibers, surface-deformed fibers, and fibers with end anchorage produce toughness indexes greater than those for smooth straight fibers at the same volume concentration, or allow similar index values to be achieved with lower fiber concentrations. For concrete containing the types of fiber with improved anchorage such as surface deformations, hooked ends, enlarged ends, or full-length crimping, index values of 5.0 for I_5 and 10.0 for I_{10} are readily achieved at fiber volumes of 1 percent or less. Such index values indicate a composite with plastic behavior after first crack that approximates the behavior of mild steel after reaching its yield point (two upper curves in Fig. 2.8). Lower fiber volumes or less effectively anchored fibers produce correspondingly lower index values (two lower curves in Fig. 2.8).

2.6-Shrinkage and creep

Tests have shown that steel fibers have little effect on free shrinkage of SFRC (Hannant 1978). However, when shrinkage is restrained, tests using ring-type concrete specimens cast around a restraining steel ring have shown that steel fibers can substantially reduce the amount of cracking and the mean crack width (Malmberg and Skarendahl 1978; Swamy and Stavrides 1979). However, compression-creep tests carried out over a loading period of 12 months showed that the addition of steel fibers does not significantly reduce the creep strains of the composite (Edgington 1973). This behavior for shrinkage and creep is consistent with the low volume concentration of fiber when compared with an aggregate volume of approximately 70 percent.

2.7-Freeze-thaw resistance

Steel fibers do not significantly affect the freeze-thaw resistance of concrete, although they may reduce the severity of visible cracking and spalling as a result of freezing in concretes with an inadequate air-void system (Aufmuth et al. 1974). A proper air-void system (AC1 201.2R) remains the most important criterion needed to insure satisfactory freeze-thaw resistance, just as with plain concrete.

2.8-Abrasion/cavitation/erosion resistance

Both laboratory tests and full-scale field trials have shown that SFRC has high resistance to cavitation forces resulting from high-velocity water flow and the damage caused by the impact of large waterborne debris at high velocity (Schrader and Munch 1976a; Houghton et al. 1978; ICOLD 1982). Even greater cavitation resistance is reported for steel fiber concrete impregnated with a polymer (Houghton et al. 1978).

It is important to note the difference between erosion caused by impact forces (such as from cavitation or from rocks and debris impacting at high velocity) and the type of erosion that occurs from the wearing action of low velocity particles. Tests at the Waterways Experiment Station indicate that steel fiber additions do not improve the abrasion/erosion resistance of concrete caused by small particles at low water velocities. This is because adjustments in the mixture proportions to accommodate the fiber requirements reduce coarse aggregate content and increase paste content (Liu 1981).

2.9-Performance under dynamic loading

The dynamic strength of concrete reinforced with various types of fibers and subjected to explosive charges; dropped weights; and dynamic flexural, tensile, and compressive loads is 3 to 10 times greater than that for plain concrete (Williamson 1965; Robins and Calderwood 1978; Suaris and Shah 1984). The higher energy required to pull the fibers out of the matrix provides the impact strength and the resistance to spalling and fragmentation under rapid loading (Suaris and Shah 1981; Gokoz and Naaman 1981).

An impact test has been devised for fibrous concrete that uses a 10-lb (4.54-kg) hammer dropped onto a steel ball resting on the test specimen. The equipment used to compact asphalt concrete specimens according to ASTM D 1559 can readily be adapted for this test; this is described in ACI 544.2R. For fibrous concrete, the number of blows to failure is typically several hundred compared to 30 to 50 for plain concrete (Schrader 1981b).

Steel fiber reinforced beams have been subjected to impact loading in instrumented drop-weight and Charpy-type systems (Suaris and Shah 1983; Naaman and Gopalaratnam 1983; Gopalaratnam, Shah, and John 1984; Gopalaratnam and Shah 1986). It was observed that the total energy absorbed (measured from the load-deflection curves) by SFRC beams can be as much as 40 to 100 times that for unreinforced beams.

CHAPTER 3-DESIGN APPLICATIONS

3.1 -Slabs

The greatest number of applications of steel fiber reinforced concrete (SFRC) has been in the area of slabs, bridge decks, airport pavements, parking areas, and cavitation/erosion environments. These applications have been summarized by Hoff (1976-1982), Schrader and Munch (1976b), Lankard (1975), Johnston (1982c), and Shah and Skarendahl (1986).

Wearing surfaces have been the most common application in bridge decks. Between 1972 and 1982, fifteen bridge deck surfaces were constructed with fiber contents from 0.75 to 1.5 volume percent. All surfaces but one were either fully or partially bonded to the existing deck, and most of these developed some cracks. In most cases, the cracks have remained tight and have not adversely affected the riding quality of the deck. A 3 in. (75 mm) thick unbonded overlay on a wooden deck was virtually crack-free after three years of traffic (ACI Committee 544, 1978). Periodic examination of the 15 projects has shown that the SFRC overlays have performed as designed in all but one case. Recently, latex-modified fiber reinforced concrete has been used successfully in seven bridge deck rehabilitation projects (Morgan 1983).

3.1.1 *Slabs on grade*-SFRC projects that are slabs on grade fall into two categories: overlays and new slabs on prepared base.

Many of the bonded or partially bonded experimental overlays placed to date without proper transverse control joints developed transverse cracks within 24 to 36 hours after placement. There are several causes for this. One is that there is greater drying shrinkage and heat release in the SFRC mixtures used because of the higher cement contents [of the order 800 lb/yd^3 (480 kg/m^3)] and the increased water demand. Recent designs have used much lower cement contents, thus reducing drying shrinkage.

It has been suggested that restrained shrinkage occurs in the overlay at a time when bond between the fiber and matrix is inadequate to prevent crack formation. In these cases, a suggested remedy is to use high-range water reducer technology and cooler placing temperatures. A study at the South Dakota School of Mines showed that drying shrinkage is reduced when the use of superplasticizers in SFRC results in a lower water-cement ratio. SFRC mixtures with w/c ratios less than 0.4 had lower shrinkage than conventional structural concrete mixtures (Ramakrishnan and Coyle 1983).

The most extensive and well monitored SFRC slab-on-grade project to date was an experimental highway overlay project in Green County, Iowa, constructed in September and October 1973 (Betterton and Knutson 1978). The project was 3.03 miles (4.85 km) long and included thirty-three 400 x 20 ft (122 x 6.1 m) sections of SFRC overlays 2 and 3 in. (50 and 75 mm) thick on badly broken pavement. Many major mixture and design variables were studied under the same loading and environmental conditions, and performance continues to be monitored.

Early observations on the Green County project indicated that the use of debonding techniques has greatly minimized the formation of transverse cracks. However, later examinations indicated that the bonded sections had outperformed the unbonded sections (Betterton and Knutson 1978). The 3 in. (75 mm) thick overlays are performing significantly better than those that are 2 in. (50 mm) thick. In the analysis of the Green County project, it was concluded that fiber content was the parameter that had the greatest impact on performance, with the higher fiber contents performing the best.

There are few well documented examples of the comparison of SFRC with plain concrete in highway slabs on grade. However, in those projects involving SFRC slabs subjected to heavy bus traffic, there is evidence that SFRC performed as well as plain concrete without fibers at SFRC thicknesses of 60 to 75 percent of the unreinforced slab thickness (Johnston 1984).

The loadings and design procedures for aircraft pavements and warehouse floors are different from those used for highway slabs. For nonhighway uses, the design methods for SFRC are essentially the same as those used for nonfiber concrete except that the improved flexural properties of SFRC are taken into account (AWI c. 1978; Schrader 1984; Rice 1977; Parker 1974; Marvin 1974; BDC 1975).

Twenty-three airport uses (Schrader and Lankard 1983) of SFRC and four experimental test slabs for aircraft-type loading have been reported. Most uses are overlays, although a few have been new slabs cast on prepared base. The airport overlays of SFRC have been constructed considerably thinner (usually by 20 to 60 percent) than a comparable plain concrete overlay would have been, and, in general, have performed well, as reported by Schrader and Lankard (1983) in a study on curling of SFRC. In those cases where comparison with a plain concrete installation was possible, as in the experimental sections, the SFRC performed significantly better.

The majority of the SFRC placements have shown varying amounts of curling at corners or edges (Schrader and Lankard 1983). The curling is similar to that evidenced by other concrete pavements of the same thickness reinforced with bar or mesh. Depending upon the amount of curling, a corner or edge crack may eventually form because of repeated bending. Thinner sections, less than 5 in. (125 mm), are more likely to exhibit curling.

The design of SFRC slabs on grade involves four considerations: (1) flexural stress and strength; (2) elastic deflections; (3) foundation stresses and strength; and (4) curl. The slab must be thick enough to accommodate the flexural stresses imposed by traffic and other loading. Since traffic-induced stresses are repetitive, a reasonable working stress must be established to insure performance under repeated loading.

In comparison with conventional concrete slabs, a fibrous concrete slab is relatively flexible due to its reduced thickness. The magnitude of anticipated elastic deflections must be assessed, because excessive elastic deflections increase the danger of pumping in the subgrade beneath the slab.

Stresses in the underlying layers are also increased due to the reduced thickness, and these must be kept low enough to prevent introduction of permanent deformation in the supporting materials.

Specific recommendations to minimize curl are available (Schrader and Lankard 1983). They include reducing the cement content, water content, and temperature of the plastic concrete, and using Type II portland cement, water reducing admixtures, and set-retarding admixtures. Other recommendations cover curing and construction practices and joint patterns.

The required slab thickness is most often based on a limiting tensile stress in flexure, usually computed by the Westergaard analysis of a slab on an elastic foundation. Selection of an appropriate allowable stress for the design is difficult without laboratory testing, because the reduction factor to account for fatigue and variability of material properties may be different for each mixture, aggregate, water-cement ratio, fiber type, and fiber content.

Parker (1974) has developed pavement thickness design curves for SFRC similar to the design curves for conventional concrete. For general SFRC, the ultimate flexural strength (modulus of rupture) is of the order 1.5 times that of ordinary concrete. A working value of 80 percent of the modulus of rupture obtained from the laboratory SFRC specimen has been conservatively suggested as a design parameter for aircraft pavements (Parker 1974). A value of two-thirds the modulus of rupture has been suggested for highway slabs.

Typical material property values for SFRC that has been used for pavements and overlays are: flexural strength = 900 to 1100 psi (6.2 to 7.6 MPa), compressive strength = 6000 psi (41 MPa), Poisson's ratio = 0.2, and modulus of elasticity = 4.0 x 10^6 psi (27,600 MPa). Typical mixtures that achieve properties in these ranges are shown in ACI 544.3R. Schrader (1984) has developed additional guidance for adapting existing pavement design charts for conventional concrete to the design of fiber reinforced concretes.

Flexural fatigue is an important parameter affecting the performance of pavements. The available data indicate that steel fibers increase the fatigue resistance of the concrete significantly. Batson et al. (1972b) found that a fatigue strength of 90 percent of the first-crack strength at 2 x 10^6 cycles to 50 percent at 10 x 10^6 cycles can be obtained with 2 to 3 percent fiber volume in mortar mixtures for nonreversal type loading. Morse and Williamson (1977), using 1.5 percent fiber volume, obtained 2 x 10^6 cycles at 65 percent of the first-crack stress without developing cracks, also for a nonreversal loading. Zollo (1975) found a dynamic stress ratio [ratio of first-crack stress that will permit 2 x 10^6 cycles to the static (one cycle) first-crack stress] for overlays on steel decks between 0.9 and 0.95 at 2 million cycles.

Generally, fatigue strengths are 65 to 95 percent at one to two million cycles of nonreversed load, as compared to typical values of 50 to 55 percent for beams without fibers. Fatigue strengths are lower for fully reversed loading. For properly proportioned high-quality SFRC, a fatigue value of 85 percent is often used in pavement design. The designer should use fatigue strengths that have been established for the fiber type, volume percent, approximate aggregate size, and approximate mortar content of the materials to be used. Mortar mixtures can accept higher fiber contents and do not necessarily behave the same as concrete mixtures.

3.1.2 *Structural floor slabs*-For small slabs of steel fiber reinforced concrete, Ghalib (1980) presents a design method based on yield line theory. This procedure was confirmed and developed from tests on one-way slabs ¾ in. thick by 6 in. wide by 20 in. long (19 x 150 x 508 mm) on an 18-in. (457-mm) span line loaded near the third points, and on two-way slabs 1.3 in. x 37.8 in. square (33 x 960 mm square) on a 35.4-in. (900-mm) span point loaded at the center. The design method applies to slabs of that approximate size only, and the designer is cautioned not to attempt extrapolation to larger slabs. Design examples given by Ghalib (1980) are for slabs about 0.78 in. (20 mm) thick.

3.1.3 *Bridge decks*-Deterioration of concrete bridge decks due to cracking, scaling, and spalling is a critical maintenance problem for the nation's highway system. One of the main causes of this deterioration is the intrusion of deicing salts into the concrete, causing rapid corrosion of the reinforcing. As discussed in Section 3.1, SFRC overlays have been used on a number of projects in an attempt to find a practical and effective method of prevention and repair of bridge deck deterioration. The ability of steel fibers to control the frequency and severity of cracking, and the high flexural and fatigue strength obtainable with SFRC can provide significant benefit to this application.

However, the SFRC does not stop all cracks, nor does it decrease the permeability of the concrete. As a consequence, SFRC by itself does not solve the problem of intrusion of deicing salts, although it may help by limiting the size and number of cracks. The corrosion of fibers is not a problem in sound concrete. They will corrode in the presence of chlorides, but their small size precludes their being a cause of spalling (Morse and Williamson 1977; Schupack 1985). See ACI 544.1R for additional data on steel fiber corrosion.

3.2- Flexure in beams

3.2.1 *Static flexural strength prediction for beams with fibers only*--Several methods have been developed to predict the flexural strength of small beams reinforced only with steel fibers (Schrader and Lankard 1983; Lankard 1972; Swamy et al. 1974). Some use empirical data from laboratory experiments. Others use the fiber bond area or the law of mixtures, plus a random distribution factor, bond stress, and fiber stress.

Equations developed by Swamy et al. (1974) have a form based on theoretical derivation with the coefficients obtained from a regression analysis of that data. Although the coefficient of correlation for the regression analysis (of the laboratory data analyzed) was 0.98, the predictions may be as much as 50 percent high for field-produced mixtures.

Concrete and mortar, a wide range of mixture proportions, fiber geometries, curing methods, and cement of two types were represented in data from several authors. The first coefficient in each equation should theoretically be 1.0. The equations are applicable only to small [4 x 4 x 12 in. (100 x 100 x 305 mm)] beams, such as those used in laboratory testing or as small minor secondary members in a structure. The designer should not attempt extrapolation to larger beams or to fiber volumes outside the normal range of the data used in the regression analysis. The equations are

first-crack composite strength, psi

$$\sigma_{cf} = 0.843\ f_r V_m + 425\ V_f\ \ell/d_f \tag{3-1}$$

ultimate composite flexural strength, psi

$$\sigma_{cu} = 0.97\ f_r V_m + 494\ V_f\ \ell/d_f \tag{3-2}$$

where

f_r = stress in the matrix (modulus of rupture of the plain mortar or concrete), psi
V_m = volume fraction of the matrix = 1 $- V_f$
V_f = volume fraction of the fibers = 1 $- V_m$
ℓ/d_f = ratio of the length to diameter of the fibers (aspect ratio)

These equations correlate well with laboratory work. However, as previously noted, if they are used to predict strengths of field placements, the predictions will generally be higher than the actual values by up to 50 percent.

3.2.2 *Static flexural analysis of beams containing bars and fibers*- A method has been developed (Henager and Doherty 1976) for predicting the strength of beams reinforced with both bars and fibers. This method is similar to the ACI ultimate strength design method. The tensile strength computed for the fibrous concrete is added to that contributed by the reinforcing bars to obtain the ultimate moment.

The basic design assumptions made by Henager and Doherty (1976) are shown in Fig. 3.1, and the equation for nominal moment M_n of a singly reinforced steel fibrous concrete beam is

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) + \sigma_t\, b\, (h - e) \left(\frac{h}{2} + \frac{e}{2} - \frac{a}{2}\right) \tag{3-3}$$

$$e = [\epsilon_s\ (\text{fibers}) + 0.003]\ c/0.003 \tag{3-4}$$

where

$$\sigma_t = 1.12\ \ell/d_f\ \rho_f\ F_{be}\ \text{(inch/pound units, psi) or} \tag{3-5}$$

$$\sigma_t = 0.00772\ \ell/d_f\ \rho_f\ F_{be}\ \text{(SI units, MPa)} \tag{3-6}$$

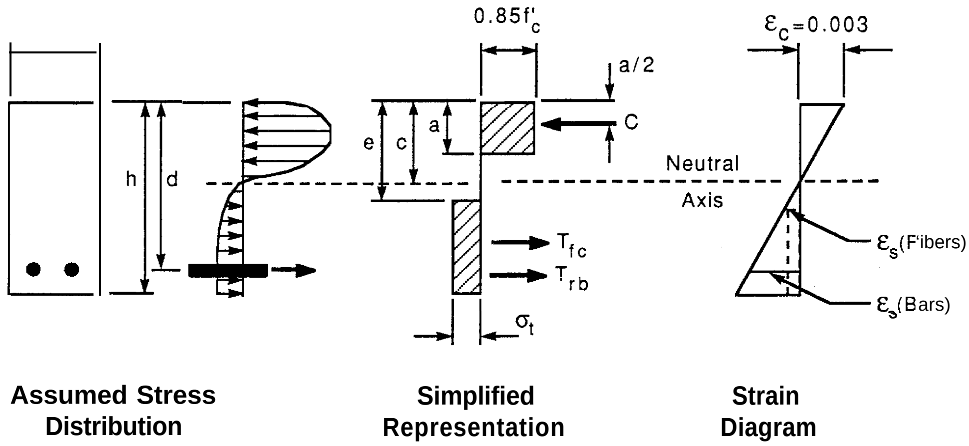

Fig. 3.1-Design assumptions for analysis of singly reinforced concrete beams containing steel fibers (Henager and Doherty 1976)

where

ℓ = fiber length
d_f = fiber diameter
ρ_f percent by volume of steel fibers
F_{be} = bond efficiency of the fiber which varies from 1.0 to 1.2 depending upon fiber characteristics
a = depth of rectangular stress block
b = width of beam
c = distance from extreme compression fiber to neutral axis found by equating the internal tension and compression forces
d = distance from extreme compression fiber to centroid of tension reinforcement
e = distance from extreme compression fiber to top of tensile stress block of fibrous concrete (Fig. 3.1)
ϵ_s = tensile strain in steel at theoretical moment strength of beam, for bars = f_y/E_s; for fibers = σ_f/E_s based on fiber stress developed at pullout (dynamic bond stress of 333 psi) (Fig. 3.1)
ϵ_c = compressive strain in concrete
f'_c = compressive strength of concrete
f_y = yield strength of reinforcing bar
A_s = area of tension reinforcement
C = compressive force
h = total depth of beam
σ_t = tensile stress in fibrous concrete
E_s = modulus of elasticity of steel
T_{fc} = tensile force of fibrous concrete = $\sigma_t\, b\, (h - e)$
T_{rb} = tensile force of bar reinforcement = $A_s f_y$

In this analysis, the maximum usable strain at the extreme concrete compression fiber is taken to be 0.003. There are some data that indicate 0.003 may be conservative. Work by Williamson (1973) and Pearlman (1979) indicates that 0.0033 may be more realistic for steel fiber concrete. Swamy and Al-Ta'an (1981) recommend 0.0035. Based on a study of plastic hinges, Hassoun and Sahebjam (1985) recommend a failure strain of 0.0035 for concrete with 1.0 percent steel fibers, and 0.004 for 1 to 3 percent fibers.

The question arises as to whether the load factors and the capacity-reduction factor for flexure used in ACI 318 are still applicable. Normally, a smaller capacity-reduction factor would be used in the calculation of design strength when concrete tension is a major part of the resisting mechanism. In this use, however, the concrete tension contributes only about 5 to 15 percent of the resisting moment, which is significant but not a major part. Additional research is needed to define the reliability of the concrete tension force before a factor can be assigned to this type of member. It would be reasonable, however, to maintain a ϕ = 0.9 for the part of the resistance attributed to the deformed bar reinforcement [first term in Eq. (3-3)], and a smaller ϕ for the concrete tension contribution [second term in Eq. (3-3)].

The ratios of the calculated moments [using Eq. (3-3)] to actual moments in test beams ranged from 1.001 to 1.017 for a series of 6 beams reported by Henager and Doherty (1976). In these tests, a SFRC mortar mixture containing 940 lb of cement/yd³ (557 kg/m³), 2256 lb (1337 kg) of ¼-in. (6-mm) maximum size aggregate, and a w/c ratio of 0.45 or less was used. The method has also been applied successfully to fiber reinforced beams using a normal cement content [420 lb/yd³ (250 kg/m³)] and to beams of fiber reinforced lightweight aggregate concrete (Henager 1977a).

Eq. (3-5) and (3-6) incorporate a factor for bond stress of the fibers; this was chosen because it correlated with these tests. The selection of 333 psi (2.3 MPa) for bond stress was based on reported values in the range of 213 to 583 psi (1.5 to 4 MPa) for smooth, straight, round, high-strength fibers with embedment lengths of ½ to 1¼ in. (12 to 32 mm) (Williamson 1974; Aleszka and Beaumont 1973; Naaman and Shah 1976). This was combined with calculations that showed that 333 psi (2.3 MPa) would not cause fracture of the fibers used in the beams.

Fiber fracture rarely occurs in SFRC flexure loading with the fiber proportions and anchorage provisions normally available and with l/d = 100 or less. In this derivation the strain in the fibers is limited to the amount that produces about 333 psi, and it does not increase because the fibers slip and pull out. It is the pullout resistance that produces the toughness characteristic of SFRC during fracture. Other methods for static flexural analysis of beams containing bars and fibers have been proposed by Schrader (197l), Williamson (1973), Swamy and Al-Ta'an (198l), and Jindal (1984). There have been studies on combined axial load and flexure that deal with the same problem of including the effect of fibers on the tension force in the concrete (Craig et al. 1984b).

3.2.3 *Beam-to-column joints*-Additional studies related to flexure have been performed on beam-to-column connections. Henager (1977b) investigated the performance of a seismic-resistant beam-column joint using steel fibers in lieu of hoops in the joint region. Longitudinal steel bars were used in both the beam and the column. Deformed steel fibers 1 ½ x 0.020 in. (38 x 0.51 mm) were used at a fiber content of 1.67 percent by volume in the joint region, an area of high shear stresses.

In comparison to a conventional joint using hoop ties at 4 in. (100 mm) on centers, the SFRC joint showed no cracking in the joint region, whereas the conventional joint showed some hairline cracking. The SFRC joint developed a maximum moment of 56.5 kip-ft (76.7 kN-m) compared to 45.9 kip-ft (62.2 kN-m) for the conventional joint. The 28-day compressive strengths were 5640 psi (38.9 MPa) for the SFRC and 5915 psi (40.8 MPa) for the conventional concrete in the joint regions. Flexural strengths were 1419 psi (9.8 MPa) for the SFRC and 450 psi (3.1 MPa) for the conventional concrete.

Craig et al. (1984a) tested 10 joints, 5 of which contained steel fibers and a reduced quantity of deformed bar hoops. He also noted considerable improvement in the joint strength, ductility, and energy absorption with the steel fibers.

3.2.4 *Flexural fatigue considerations*-Batson et al. (1972b) recommended that 67 percent of the first-crack stress be used for 10^6 cycles of load in conventionally reinforced SFRC beams. Schrader (1971) has shown that the post-fatigue load-carrying capacity of SFRC beams is improved, but that the presence of conventional reinforcing bars overshadows the fatigue and static strength improvements obtained when comparing SFRC beams to beams with no conventional reinforcing.

Kormeling, Reinhardt, and Shah (1980) tested conventionally reinforced concrete beams with and without fibers in fatigue loading up to 10 million cycles. It was observed that the addition of fibers to conventionally reinforced concrete beams increased the fatigue life and decreased deflections and crack widths for a given number of dynamic cycles. The beneficial effect of fibers decreased with increasing volume of conventional reinforcement.

3.3-Shear in beams

There are considerable laboratory data indicating that fibers substantially increase the shear (diagonal tension) capacity of concrete and mortar beams. Steel fibers show several potential advantages when used to supplement or replace vertical stirrups or bent-up steel bars. These advantages are: (1) the fibers are randomly distributed through the volume of the concrete at much closer spacing than can be obtained with reinforcing bars; (2) the first-crack tensile strength and the ultimate tensile strength are increased by the fibers; and (3) the shear-friction strength is increased.

It is evident from a number of tests that stirrup and fiber reinforcement can be used effectively in combination. However, although the increase in shear capacity has been quantified in several investigations it has not yet been used in practical applications. This section presents the results of some of the studies dealing with the effect of steel fibers on shear strength in beams and slabs. It is important to identify the type and size of fiber upon which the design is based.

Batson et al. (1972a), using mortar beams 4 x 6 x 78 in. (100 x 150 x 2000 mm), conducted a series of tests to determine the effectiveness of straight steel fibers as web reinforcement in beams with conventional flexural reinforcement. In tests of 96 beams, the fiber size, type, and volume concentration were varied, along with the shear-span-to-depth ratio a/d, where a = shear span (distance between concentrated load and face of support) and d = the depth to centroid of reinforcing bars. (Shear capacity of rectangular beams may be considered a function of moment-to-shear ratio a/d or M/Vd.) Third-point loading was used throughout the test program.

It was found that, for a shear-span-to-depth ratio of 4.8, the nonfiber beams failed in shear and developed a shear stress at failure of 277 psi (1.91 MPa). For a fiber volume percent of 0.88, the average shearing stress at failure was 310 psi (2.14 MPa) with a moment-shear failure; for 1.76 volume percent, 330 psi (2.28 MPa) with a moment failure; and for 2.66 volume percent, 352 psi (2.43 MPa), also with a moment failure. The latter value represents an increase of 27 percent over the nonfiber beams. The shear stress at failure for beams with #3 [3/8-in. (9.5-mm) diameter] stirrups at 2-in. (50-mm) spacing in the outer thirds averaged 315 psi (2.17 MPa). All shearing stresses were computed by the equation $v = VQ/Ib$.

It was found that as the shear-span ratio decreased and fiber volume increased, higher shear stresses were developed at failure. For example, for an a/d of 3.6 and a volume percent of fiber of 0.88, the shear stress at failure was 444 psi (3.06 MPa) with a moment failure; for an a/d of 2.8 and a fiber volume percent of 1.76, the shear stress at failure was 550 psi (3.79 MPa) and a moment failure.

Paul and Sinnamon (1975) studied the effect of straight steel fibers on the shear capacity of concrete in a series of seven tests similar to those of Batson et al. (1972a). The objective was to determine a procedure for

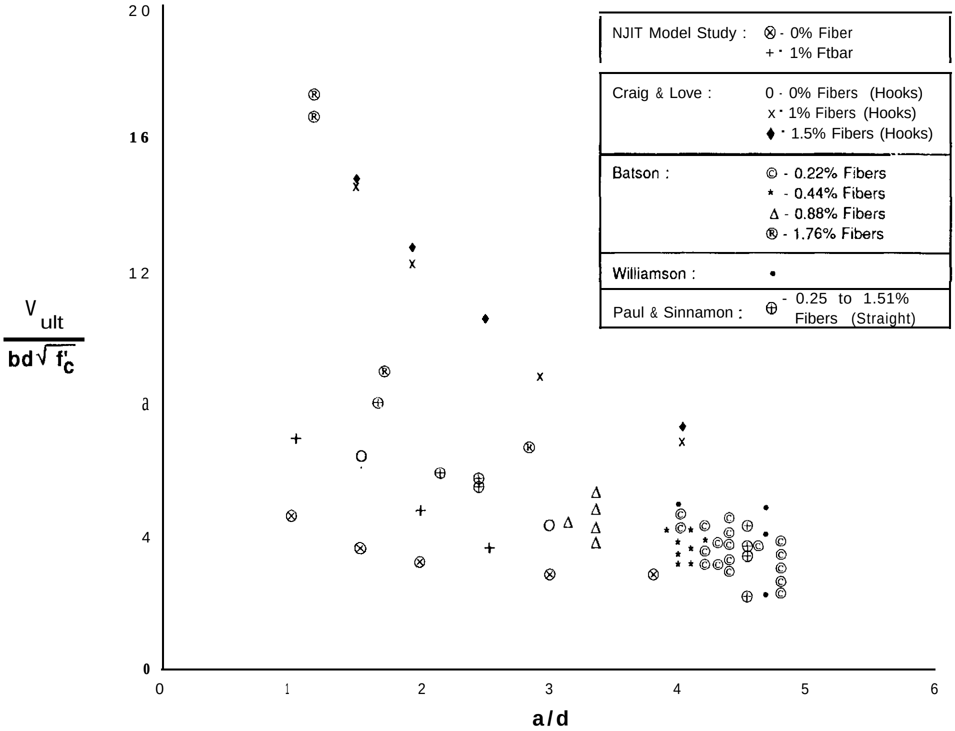

Fig. 3.2-Shear behavior of reinforced fibrous concrete beams

predicting the shear capacity of segmented concrete tunnel liners made with steel fiber reinforced concrete. Their results agreed closely with Batson, especially for beams with similar *a/d* ratios.

Williamson (1978), working with conventionally reinforced beams 12 x 21.5 in. x 23 ft (305 x 546 x 7010 mm), found that when 1.66 percent by volume of straight steel fibers were used in place of stirrups, the shear capacity of the beams was increased 45 percent over a beam without stirrups. Nevertheless, the beams failed in shear. This is consistent with the results of other investigators. When steel fibers with deformed ends were used (1.1 percent by volume), the shear capacity was increased by 45 to 67 percent and the beams failed in flexure.

Williamson (1978) concluded that, based upon the use of steel fibers with deformed ends, steel fibers can increase the shear strength of concrete beams enough to prevent catastrophic diagonal tension failure and to force the beam to fail in flexure. In his report, Williamson (1978) presents an analysis showing that steel fibers can present an economical alternative to the use of stirrups in reinforced concrete design.

Tests of crimped-end fibers have shown considerable increase in the shear capacity of reinforced concrete in other studies. Some of the tests at the New Jersey Institute of Technology (Craig 1983) have shown increases of more than 100 percent. Twelve full-scale test beams with 1.0 and 1.5 percent by volume of 0.020 x 1.18 in. (0.5 x 30 mm) long crimped-end fibers were tested with the following span-to-depth ratios: *a/d* = 1.0, 1.5, 2.0, 2.5, and 3.0. The beams had a 6 x 12 in. (150 x 300 mm) section. The increases in shear capacity for the 1.0 and 1.5 percent fiber content with *a/d* = 1.5 were 130 and 140 percent, respectively. Similarly, the increase at *a/d* = 3.0 was 108 percent for 1.0 volume percent of fiber. The combination of stirrups and fibers showed slow and controlled cracking and better distribution of tensile cracks, and minimized the penetration of shear cracks into the compression zone.

It was also found that when fibers with crimped ends were the only shear reinforcement, there was a significant decrease in diagonal tension cracking in the beams. Fig. 3.2 shows the results of the tests reported by Craig (1983) and compares them with other test results.

Bollano (1980) investigated the behavior of steel fibers as shear reinforcement in two-span continuously reinforced concrete beams. These tests indicate the behavior in shear for the common range of *M/Vd* ratios for negative moment regions (*M/Vd* = **2** to 3, equivalent to *a/d* for simple beams). It is generally assumed that the *M/Vd* concept can be used equally well in simply supported and continuous beams, but this is not entirely true for the beams investigated. The *a/d* ratio was 4.8 and the *M/Vd* ratio was 3.0. The regular reinforced concrete beam $V/bd\sqrt{f'_c}$ ranged from 3 to 4, whereas this parameter for the beams with straight and crimped-end fibers ranged from 5 to 8, showing significant improvement with the addition of fibers.

Criswell (1976) conducted a number of different shear tests, all of which demonstrated an increased shear capacity with the use of steel fibers. All of his tests were made with concrete containing 1.0 percent by volume of straight fibers. The results of four shear-friction specimens showed a 20 percent increase in shear strength; bolt pullout tests showed a shear strength in excess of 64 percent greater than that for the nonfiber concrete; slab-column connection specimens developed shearing strengths 27 percent greater than the nonfiber specimens; and beam-column shear tests resulted in shear strengths up to 60 percent greater.

Sharma (1986) tested 7 beams with steel fiber reinforcement, of which 4 also contained stirrups. The fibers had deformed ends. Based on these tests and those by Batson et al (1972a) and Williamson and Knab (1975), he proposed the following equation for predicting the average shear stress v_{cf} in the SFRC beams. (In the equation that follows, a typographical error in Sharma's 1986 paper has been corrected.)

$$v_{cf} = \frac{2}{3} f_t' \left(\frac{d}{a}\right)^{0.25} \tag{3-7}$$

where f_t' is the tensile strength of concrete obtained from results of indirect tension tests of 6 x 12 in. (150 x 300 mm) cylinders, and *d/a* is the effective depth-to-shear-span ratio. Straight, crimped, and deformed-end fibers were included in the analysis and the average ratio of experimental to calculated shear stress was 1.03 with a mean deviation of 7.6 percent. The influence of different fiber types and quantities is considered through their influence on the parameter f_t'. The proposed design approach follows the method of ACI 318 for calculating the contribution of stirrups to the shear capacity, to which is added the resisting force of the concrete calculated from the shear stress given by Eq. (3-7).

An additional design procedure for shear and torsion in composite reinforced concrete beams with fibers has been published by Craig (1986).

3.4-Shear in slabs

The influence of steel fiber reinforcement on the shear strength of reinforced concrete flat plates was investigated by Swamy et al. (1979) in a test series on four slabs with various fiber contents (0, 0.6, 0.9, and 1.2 percent by volume). The slabs were 72 x 72 x 5 in. (1830 x 1830 x 125 mm) with load applied through a square column stub 6 x 6 x 10 in. (150 x 150 x 250 mm). All slabs had identical tension and compression reinforcement, and the steel fibers had crimped ends and were 0.02 x 2 in. (0.5 x 50 mm) long. The shear strength increases were 22, 35, and 42 percent for the 0.6, 0.9, and 1.2 percent by volume fiber contents, respectively.

3.5-Shotcrete

Steel fiber shotcrete has been used in the construction of dome-shaped structures using the inflation/foam/shotcrete process (Williamson et al. 1977; Nelson and Henager 1981). Design of the structures follows the conventional structural design procedures for concrete domes, taking into account the increased compressive, shear, and flexural properties of fibrous concrete.

This material is also used for underground support and linings, rock slope stabilization, repair of deteriorated concrete, etc. (Kobler 1966; Shah and Skarendahl 1986; Morgan and McAskill 1984). A research effort carried out in a side chamber of an Atlanta subway station to examine shotcrete support in loosening rock is reported by Fernandez-Delgado et al. (1981).

A significant quantity of steel fiber reinforced shotcrete has been used throughout the world, and a state-of-the-art report has been prepared by ACI Committee 506 (ACI 506.1R). That report also contains information on material properties, application procedures, and mixtures.

3.6-Cavitation erosion

Failure of hydraulic concrete structures is often precipitated by cavitation-erosion failure of the concrete. SFRC was used to repair severe cavitation-erosion damage that occurred in good quality conventional concrete after relatively short service at Dworshak, Libby, and Tarbella Dams (ICOLD 1982; Schrader and Munch 1976a). All three are high-head structures capable of large flows and discharge velocities in excess of 100 fps (30.5 mps).

At Libby and Dworshak, both the outlet conduits and stilling basins were repaired. At Tarbella, fiber concrete was used as topping in the basin and ogee curve leading from the outlet conduit to the basin. All three projects have performed well since the repairs. It should be noted, however, that while SFRC improves resistance to erosion from cavitation, it does not improve resistance to erosion from abrasion or scouring (see Section 2.8).

3.7-Additional applications

There are several applications of SFRC that have involved a considerable volume of material, but which do not have well defined design methods specifically for SFRC. Among these are fence posts, sidewalks, embankment protection, machinery foundations, machine tool frames, manhole covers, dolosse, bridge deck expansion joints (nosings at joints to improve wear and impact resistance), dams, electric power manholes, ditch linings, mine cribbing, liquid storage tanks, tilt-up wall construction, and thin precast members (see also Shah and Batson 1987).

CHAPTER 4-REFERENCES

4.1 -Specified and/or recommended references

The standards of the American Society for Testing and Materials and the standards and reports of the American Concrete Institute referred to in this report are listed below with their serial designation, including the year of adoption or revision. The standards and reports listed were the latest editions at the time this report was prepared. Since some of these publications are revised frequently, generally in minor details only, the user of this report should check directly with the sponsoring group to refer to the latest edition.

American Concrete Institute

201.2R-77 Reapproved 1982	Guide to Durable Concrete
223-83	Standard Practice for the Use of Shrinkage-Compensating Concrete
318-83 (Revised 1986)	Building Code Requirements for Reinforced Concrete
506R-85	Guide to Shotcrete
506.1R-84	State-of-the-Art Report on Fiber Reinforced Shotcrete
506.2-77	Standard Specification for Materials, Proportioning, and Application of Shotcrete
544.1 R-82 (Reapproved 1986)	State-of-the-Art Report on Fiber Reinforced Concrete
544.2R-78 (Revised 1983)	Measurement of Properties of Fiber Reinforced Concrete
544.3R-84	Guide for Specifying, Mixing, Placing and Finishing Steel Fiber Reinforced Concrete
549R-82	State-of-the-Art Report on Ferrocement

ASTM

A 820-85	Standard Specification for Steel Fibers for Use in Fiber Reinforced Concrete
C 78-84	Standard Test Method for Flexural Strength of Concrete (Using Simple Beam with Third-Point Loading)
C 143-78	Standard Test Method for Slump of Portland Cement Concrete
C 157-80	Standard Test Method for Length Change of Hardened Cement Mortar and Concrete
C 666-84	Standard Test Method for Resistance of Concrete to Rapid Freezing and Thawing
C 995-86	Standard Test Method for Time of Flow of Fiber-Reinforced Concrete Through Inverted Slump Cone
C 1018-85	Standard Test Method for Flexural Toughness and First Crack Strength of Fiber-Reinforced Concrete (Using Beam with Third-Point Loading)
D 1559-82	Standard Test Method for Resistance to Plastic Flow of Bituminous Mixtures Using Marshall Apparatus

The above publications may be obtained from the following organizations:

American Concrete Institute
P. 0. Box 19150
Detroit, MI 48219-0150

ASTM
1916 Race Street
Philadelphia, PA 19103

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CHAPTER 5 — NOTATION

a = depth of rectangular stress block
a = shear span, distance between concentrated load and face of support
A_s = area of tension reinforcement bars
b = width of beam

b_w = web or width of a rectangular beam
c = distance from extreme compression fiber to neutral axis
C = compressive force
d = distance from extreme compression fiber to centroid of tension reinforcement
d_f = fiber diameter (for a noncircular fiber, an equivalent fiber diameter is the diameter of a circle with the same area as the fiber)
e = distance from extreme compression fiber to top of tensile stress block of fibrous concrete
E = modulus of elasticity
E_s = modulus of elasticity of steel
f'_c = compressive strength of concrete
f_{ct} = splitting tensile strength
f_r modulus of rupture
f_y = yield strength of reinforcing bar
F_{be} = bond efficiency factor
h = total depth of beam
I = moment of inertia of section
M_n = nominal moment capacity of section
M_u = factored moment at beam section
ℓ = fiber length
ℓ/d_f = aspect ratio = fiber length/fiber diameter
Q = first statical moment of an area about the neutral axis
T_{fc} = tensile force of fibrous concrete = $\sigma_1 b\,(h - e)$
T_{rb} = tensile force of bar reinforcement = $A_s f_y$
v = fiber volume concentration or volume fraction (not percentage)
v = shear stress at section
v_{cf} = average shear stress in SFRC beam
V = shear force at section
V_c = nominal shear strength provided by concrete
V_f = volume fraction of fibers $(1 - V_m)$
V_m = volume fraction of the matrix $(1 - V_f)$
V_u = factored shear force at beam section
ϵ_c = compressive strain in concrete
ϵ_s = tensile strain in steel
σ_{cf} = first crack composite flexural strength
σ_{cu} = ultimate composite flexural strength
σ_f = tensile stress in fiber
σ_t = tensile stress in fibrous concrete
τ_d = dynamic bond stress between fiber and matrix
ρ_f = percent by volume of fibers
ρ_w = $A_s/b_w d$
ϕ = capacity reduction factor

This report was submitted to letter ballot of the committee and was approved in accordance with ACI balloting requirements.