Guide to Design of Reinforced Two-Way Slab Systems

Reported by Joint ACI-ASCE Committee 421

ACI 421.3R-15

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First Printing
October 2015
ISBN: 978-1-942727-56-9

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This guide presents analysis methods, design procedures, slab reinforcement and detailing practices, and strength and serviceability considerations, as well as information for the resistance to lateral forces for slab-column frames. It also covers the design for flexure and shear and torsion, as well as the effect of openings. Both two-way nonprestressed slabs and post-tensioned slabs are included.

Keywords: analysis method; deflection; direct design; flat plates; flat slabs; post-tension; reinforcement; shear; shearhead; slab-column frame; two-way slabs.

CONTENTS

ACI 421.3R-15 was adopted and published October 2015.

CHAPTER 1—INTRODUCTION AND SCOPE

1.1—Introduction: history of two-way slab system

Two-way flat slab construction in the United States evolved, and was invented and patented, in the early 1900s (Cohen and Heun 1979). Early two-way flat slab construction was built and subjected to load tests in place and scaled models were later tested in laboratories. While the amount of reinforcement in slab construction varied dramatically, flat slab systems were found to be economical for heavy live load occupancy. As the number of flat slab projects increased steadily worldwide, design rules were established and formalized (Sozen and Seiss 1963).

Prior to the 1950s, two-way waffle slabs and two-way flat slabs were designed and constructed with column capitals and some with drop panels. The hollow tile and concrete slab is a type of waffle slab that dates back to at least 1918 (Gamble et al. 1964). Column capitals were used to increase slab shear strength and drop panels to reduce the flexural reinforcement over columns, which allowed for thinner slabs. In the post-1970s era, field labor to construct formwork for column capitals and drop panels became costly; the introduction of reusable forms led to construction of flat plates, which are two-way flat slabs without column capitals or drop panels.

The lift-slab system for multistory construction was popular in the 1960s and 70s, but is no longer commonly used. The slabs were cast in a stack at ground level, post-tensioned, and then lifted to their final elevations using jacks lifting on steel collars embedded in the slabs.

Draped post-tensioning can be designed to balance part of the gravity loads. Combining unbonded post-tensioned tendons and nonprestressed reinforcement results in reduced slab thickness. In addition, the use of nonprestressed reinforcement supplements prestressed tendons to meet the required nominal strength and control slab cracking.

1.2—Scope

The performance record of various two-way slab systems is well established based on results of extensive tests and practical construction improvements in the twentieth century. The ACI Building Code permits design of slab systems, both nonprestressed and post-tensioned, based directly on fundamental principles of structural mechanics that satisfy equilibrium and compatibility. This guide provides classic solutions based on linearly elastic continuum, as well as prescriptive procedures used in common practice for analysis and design of slab systems. The fundamental principles in this guide are applicable to all planar structural slab systems subjected to gravity loads and, in certain conditions, those combined with lateral forces.

This guide addresses recommended practice in the selection and distribution of flexural reinforcement, and guidelines to transmit loads from slabs to columns by flexure, torsion, and shear. Detailing practices for post-tensioned two-way slabs are found in ACI 423.3R-05. This guide also discusses aspects and parameters where two-way slabs without beams are incorporated in ordinary or intermediate moment frames with ductile detailing and toughness.

While two-way slab systems have more than 100 years of service history, various practical refinements and research programs continue to develop new materials and technologies that support sustainable construction of two-way slabs.

CHAPTER 2—NOTATION AND DEFINITIONS

2.1—Notation

A_{cf} = larger gross cross-sectional area of the slab-column strips in the two orthogonal equivalent frames intersecting at a column in a two-way slab, ft^2 (m^2)

A_{sb} = area of reinforcement through the column core used as integrity reinforcement

b_1 = dimension of the critical section b_o measured in the direction of the span for which moments are determined, in. (mm)

b_2 = dimension of the critical section b_o measured in the direction perpendicular to b_1, in. (mm)

b_e = effective slab width, in. (mm)

b_o = perimeter of critical section at $d/2$ from face of support, in. (mm)

C = cross-sectional constant to define torsional properties of slab and beam

c_1 = dimension of rectangular or equivalent rectangular column, capital, or bracket measured in the direc-

tion of the span for which moments are being determined, in. (mm)
c_2 = dimension of rectangular or equivalent rectangular column, capital, or bracket measured in the direction perpendicular to c_1, in. (mm)
c_t = distance from the interior face of the column to slab edge measured parallel to c_1, but not exceeding c_1, in. (mm)
d = distance from extreme compression fiber to the centroid of tension reinforcement, in. (mm)
E_{cb} = modulus of elasticity of beam concrete, psi (MPa)
E_c = modulus of elasticity of slab concrete, psi (MPa)
f_c' = specified compressive strength of concrete, psi (MPa)
f_{pc} = average compressive stress in the two directions at centroid of concrete cross section after allowing for all prestress losses, psi (MPa)
f_y = specified yield stress of reinforcement, psi (MPa)
g = distance between adjacent stirrup legs or studs, measured in a parallel direction to a column face
h = slab thickness, in. (mm)
I_b = moment of inertia of gross section of beam about centroidal axis, in.^{4} (mm^4)
I_s = moment of inertia of gross section of slab about centroidal axis defined for calculating α_f and β_t, in.^{4} (mm^4)
K_c = stiffness of columns based on moment of inertia at any cross section outside the joint
K_{ec} = stiffness of equivalent column
K_{FP} = modification factor accounting for reduction in joint confinement at exterior connections
K_t = torsional stiffness
ℓ_1 = length of span in direction that moments are being determined, measured center-to-center of supports, in. (mm)
ℓ_2 = length of span in direction perpendicular to ℓ_1, measured center-to-center of supports, in. (mm)
ℓ_3 = distance measured from the column centerline to the edge of the slab, in. (mm)
ℓ_n = length of clear span measured face-to-face of supports, in. (mm)
ℓ_t = span of member under load test, taken as the shorter span for two-way slab systems, in. (mm); span is the smaller of: a) distance between centers of supports; and b) clear distance between supports plus thickness h of member.
M_o = total factored static moment, in.-lb (kNm)
M_{sc} = portion of slab factored moment balanced by support moment, in.-lb (kNm)
N_c = resultant tensile force acting on the portion of the concrete cross section that is subjected to tensile stresses due to the combined effects of service loads and effective prestress, lb (N)
q_{Du} = factored dead load per unit area, lb/ft^2 (kPa)
q_{Lu} = factored live load per unit area, lb/ft^2 (kPa)
q_u = factored load per unit area, lb/ft^2 (kPa)
x = shorter overall dimension of rectangular part of cross section, in. (mm)
y = longer overall dimension of rectangular part of cross section, in. (mm)
V_c = nominal shear strength provided by concrete, lb (N)
V_p = vertical component of all effective prestress forces crossing the critical section, lb (N)
V_{se} = unfactored shear force, but not less than twice the unfactored dead load shear, lb (N)
V_{ug} = factored shear force on the slab critical section for two-way action due to gravity loads, lb (N)
α_f = ratio of flexural stiffness of beam section to flexural stiffness of a width of slab bounded laterally by centerlines of adjacent panels (if any) on each side of the beam
α_{fm} = average value of α_f for all beams on edges of a panel
α_{f1} = α_f in direction of ℓ_1
α_{f2} = α_f in direction of ℓ_2
α_s = constant used to compute V_c in slabs
β = ratio of long side-to-short side of the column, concentrated load, or reaction area
β_p = factor used to compute V_c in prestressed slabs
β_t = ratio of torsional stiffness of edge beam section to flexural stiffness of a width of slab equal to span length of beam, center-to-center of supports
γ_f = factor used to determine the unbalanced moment transferred by flexure at slab-column connections
γ_v = factor used to determine the unbalanced moment transferred by eccentricity of shear at slab-column connections
ρ_b = ratio of A_s to b_d at balanced condition when concrete and reinforcement both reach their respective yield strain
$\sum A_{sb}$ = total area of reinforcing steel passing through the column core, summed on all four sides of the column of an interior column
χ = ℓ_2/ℓ_1

2.2—Definitions

ACI provides a comprehensive list of definitions through an online resource, "ACI Concrete Terminology," https://www.concrete.org/store/productdetail.aspx?ItemID=CT13. Definitions provided herein complement that source.

design story drift—design displacement of one level, or floor, relative to the level above or below.

design story drift ratio—design story drift divided by the story height.

headed shear stud reinforcement—individual headed studs or groups of studs with anchorage provided by a head at each end or a common base rail consisting of a steel plate or shape.

lateral-force-resisting system—portion of the structure composed of members designed to resist forces related to wind or earthquake effects.

podium—a thick slab that supports a light frame building above parking or commercial levels.

shear cap—a projection below the slab used to increase the slab shear strength.

slab-on-girder system—a grid of girders in both plan directions with a solid slab spanning between girders; may include beams between girders in both directions.

slab panel—a slab bounded by column, beam, or wall centerlines on all sides.

two-way slab—slab with or without beams that meets a particular aspect ratio.

two-way wide-band system—slab with paneled ceiling with shallow and wide beams spanning between columns in each direction.

waffle slab—two-way concrete joist construction using prefabricated pan (dome) forms.

CHAPTER 3—ANALYSIS METHODS

3.1—General

3.1.1 *Fundamental principles of structural mechanics*—Analysis of two-way slab can be based directly on fundamental principles of structural mechanics, provided it can be demonstrated explicitly that all strength and serviceability criteria in Chapters 4 and 6 of this guide are satisfied. Based on theory of elasticity, slab design should satisfy force equilibrium and strain compatibility. Slab analysis can be achieved through the use of classic solutions based on linearly elastic continuum theories; numerical solutions based on discrete elements; yield-line analyses; or strip method analyses, including in all cases evaluation of the stress conditions around the supports in relation to shear and torsion, as well as flexure. Regardless of analysis method, deviations in physical dimensions of the slab from common practice should be justified on the basis of knowledge of the expected loads, reliability of the calculated stresses, and deformations of the structure.

Analysis of a two-way slab system should consider the aspect ratio of each slab panel and the relative stiffness of the slab panels, supporting beams (if any), and supporting columns or walls. Analysis and design of two-way slab systems using the Direct Design Method (DDM) and Equivalent Frame Method (EFM) are discussed in 3.2.

3.1.2 *Slab stiffness*—During the life of a structure, construction loads, ordinary occupancy loads, anticipated overloads, and volume changes can cause cracking of slabs. Excessive cracking exposes concrete to moisture infiltration, which can cause corrosion of reinforcement and deterioration of structural elements. Under sustained gravity loads, cracking can lead to large vertical deflection resulting in damage to nonstructural elements.

Cracking reduces stiffness of the slabs, and increases lateral displacement when lateral forces act on the structure. Cracking of slabs should be considered in stiffness assumptions so drift caused by wind or earthquake is not grossly underestimated. Conservatively, analysis of slab-column connections can use a stiffness reduction factor that provides higher reduction in stiffness to ensure lateral displacement and that the design forces are not underestimated. Refer to 7.2.2 for further discussions.

3.1.3 *Total factored static moment*—Total factored static moment M_o for a span of a rectilinear interior panel may be determined in a strip bounded laterally by the centerline of a panel on each side of the centerline of supports. The absolute sum of positive and average negative factored moments in each direction should not be less than

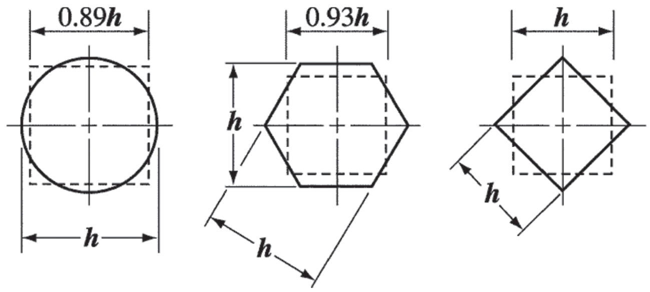

Fig. 3.1.3—Example of equivalent square section for supporting members.

$$M_o = \frac{q_u \ell_2 \ell_n^2}{8} \tag{3.1.3}$$

In the case of an edge panel, use $0.5\ell_2$, where ℓ_2 *is* the transverse span.

Equation (3.1.3) follows directly from Nichols (1914) with the simplifying assumption that the reactions are concentrated along the faces of the support perpendicular to the span considered. In general, the designer will find it expedient to calculate the static moment for two adjacent half panels, which includes a column strip with a half middle strip along each side. The clear span ℓ_n is taken as the distance between columns, capitals, brackets, or walls. The value of ℓ_n used in Eq. (3.1.3) should not be less than $0.65\ell_1$. Circular or regular polygon-shaped supports should be treated as square supports with the same area (Fig. 3.1.3).

3.2—Analysis methods

Both the DDM and EFM are based on analysis of the results of an extensive series of tests (Jirsa et al. 1963) and the well-established performance record of various slab systems. This guide covers these two methods in depth.

3.2.1 *Direct Design Method (DDM)*—The DDM consists of a set of rules for distributing moments to slab and beam sections to satisfy strength requirements. The DDM can be used for slab systems with regular column layouts subjected to gravity loads only. Three fundamental steps are involved:

1) Determination of M_o

2) Distribution of M_o along each span to supports and midspan for negative factored moments and positive factored moment, respectively

3) Distribution of the negative and positive factored moments transversely to the column and middle strips and to the beams, if any. (The same distribution of moments to column and middle strips is also used in EFM, which is discussed in 3.2.2.6.)

3.2.1.1 *Limitations*—The DDM was developed from considerations of theoretical procedures for the determination of moments in slabs with and without beams, requirements for simple analysis and construction procedures, and precedents supplied by performance of slab systems. Consequently, based on ACI 318-14, Section 8.10.1.1, the analysis of slab systems using the DDM should conform to the following limitations:

a) Slab system has a minimum of three continuous spans in each direction;

b) Panels are rectangular, with a ratio of longer-to-shorter panel span taken from center-to-center of each panel not greater than 2;

c) Successive span lengths center-to-center of supports in each direction do not differ by more than one-third the longer span;

d) Offset of columns by a maximum of 10 percent of the span (in direction of offset) from either axis between centerlines of successive columns is permitted;

e) All loads are due to gravity only and uniformly distributed over an entire panel and unfactored live load does not exceed two times unfactored dead load;

f) For a panel with beams between supports on all sides, Eq. (3.2.1.1a) should be considered for beams in the two perpendicular directions:

$$0.2 \le \frac{\alpha_{f1}\ell_2^{\,2}}{\alpha_{f2}\ell_1^{\,2}} \le 5.0 \qquad (3.2.1.1a)$$

where α_{f1} and α_{f2} are calculated in accordance with Eq. (3.2.1.1b)

$$\alpha_f = \frac{E_{cb}I_b}{E_{cs}I_s} \qquad (3.2.1.1b)$$

g) Moment redistribution as permitted by ACI 318-14, Section 6.6.5, is not applicable for slab systems analyzed by the DDM;

h) Variations from these limitations are permitted if demonstrated by analysis from 3.1.1 of this guide.

Rules given for the DDM assume that the slab system at the first interior support is neither fixed against rotation nor discontinuous. The primary reason for limitation a), which requires a minimum of three spans, is that the magnitude of the negative moments at the interior support in a structure with only two continuous spans is higher by approximately 10 percent. Under limitation b), when the ratio of the two spans (long span/short span) of a panel exceeds 2, the slab resists the moment in the shorter span essentially as a one-way slab.

Limitation c) relates to the possibility of developing negative moments beyond the point where negative moment reinforcement is terminated, as discussed in a subsequent section. Limitation d) permits columns to be offset within specified limits from a regular rectangular array. A cumulative total offset of 20 percent of the span is established as the upper limit when using the DDM.

Limitation e) is included because the DDM is based on tests (Jirsa et al. 1969) for uniform gravity loads and resulting column reactions determined by statics. Lateral loads such as wind or seismic require a frame analysis. Inverted foundation mats analyzed as two-way slabs involve application of known column loads. Therefore, even where the soil reaction is assumed to be uniform, a frame analysis should be performed. The limit of applicability of the DDM for ratios of live-to-dead load was reduced in ACI 318-95 from 3 to 2. Because in most slab systems the live-to-dead load ratio is less than 2, it is unnecessary to check the effects of pattern loading. For a slab system supporting a nonmovable load (such as a water reservoir in which the load on all panels is expected to be the same), the designer need not consider the live load imitation e) because there is no need to consider pattern loading.

The elastic distribution of moments will deviate significantly from those assumed in the DDM unless the stiffness under limitation f) is met. Limitation g) forbids the use of moment redistribution as permitted by ACI 318-14, Section 6.6.5, which is not intended for use where approximate values for bending moments are used. For the DDM, 10 percent modification is allowed as described in 3.2.2.5 of this guide.

Limitation h) permits a designer to use the DDM even when the structure does not fit the limitations in this section, provided it can be shown by analysis that the particular limitation does not apply to that structure.

3.2.1.2 *Distribution of total factored moments*—As stated in 3.1.3, the clear span is based on distance between faces of supports. If a supporting member does not have a rectangular cross section or if the sides of the rectangle are not parallel to the spans, it should be treated as a square support having the same area, as illustrated in Fig. 3.1.3.

In an interior span, for M_o, calculated based on Eq. (3.1.3) should be distributed as follows:

a) Negative factored moment: 0.65

b) Positive factored moment: 0.35

The M_o, in an end span, should be distributed to slab midspan and supports as shown in Table 3.2.1.2.

Table 3.2.1.2—Distribution coefficient of end span total factored static moment, M_o

	(1)	(2)	(3)	(4)	(5)
	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports		Exterior edge fully restrained
Interior negative factored moment	0.75	0.70	0.70	0.70	0.65
Positive factored moment	0.63	0.57	0.52	0.50	0.35
Exterior negative factored moment	0	0.16	0.26	0.30	0.65

The moment coefficients for an end span are based on the equivalent column stiffness expressions (Corley et al. 1961; Jirsa et al. 1963; Corley and Jirsa 1970). The coefficients for

Table 3.2.1.3a—Distribution in percent of interior negative factored moment to column strips

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	75	75	75
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

Note: Linear interpolations can be made between values shown.

an unrestrained edge would be used, for example, if the slab is simply supported on a masonry or concrete wall. Those for a fully restrained edge would apply if the slab is constructed integrally with a concrete wall having a flexural stiffness so large compared to that of the slab that little rotation occurs at the slab-to-wall connection. Note that Table 3.2.1.2 is a major simplification of values that is computed on the basis of relative stiffnesses (ACI 318-77) and that, in some cases, these moments could deviate substantially larger from theoretic values. This is especially true for the exterior negative moment case because there are no limits on column stiffness other than pinned or fixed.

For other than unrestrained or fully restrained edges, coefficients in Table 3.2.1.2 were selected to be near the upper bound of the range for positive moments and interior negative moments. As a result, exterior negative moments were usually closer to a lower bound. The exterior negative moment strength for most slab systems is governed by spacing of minimum reinforcement to control cracking. The final coefficients in the table have been adjusted so that the absolute sum of the positive and average negative moments equal M_o.

For two-way slab systems with beams between supports on all sides (two-way slabs), moment coefficients of Column (2) of Table 3.2.1.2 apply. For slab systems without beams between interior supports (flat plates and flat slabs), the moment coefficients of Columns (3) or (4) apply, without or with an edge (spandrel) beam, respectively. Design should be based on the larger of the two interior negative factored moments on either side of a common column or other type of support. If an analysis is made to distribute unbalanced moments, flexural stiffness could be obtained on the basis of the gross concrete section of the members involved.

The gravity load moment transferred by eccentricity within the critical section for shear between slab and edge column should be $0.3M_o$. Proportion edge beams or edges of slab to resist exterior negative factored moments. Torsional stresses caused by the moment assigned to the slab should be investigated.

3.2.1.3 *Factored moments in column strips*—The rules for assigning moments to the column strips, beams, and middle strips are based on studies (Gamble 1972) of moments in linearly elastic slabs with different beam stiffness tempered by the moment coefficients that have been used successfully. Column strips should be proportioned to resist the portions in percent tabulated in Table 3.2.1.3a for interior negative factored moments, and in Table 3.2.1.3b for exterior negative factored moments.

In Table 3.2.1.3b, linear interpolations can be made between values shown, where β_t is calculated in Eq. (3.2.1.3a) and C is calculated in Eq. (3.2.1.3b).

Table 3.2.1.3b—Distribution in percent of exterior negative factored moment to column strips

ℓ_2/ℓ_1		0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	75	75	75
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	90	75	45

Table 3.2.1.3c—Distribution in percent of positive factored moment to column strips

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	60	60	60
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

Note: Linear interpolations can be made between values shown.

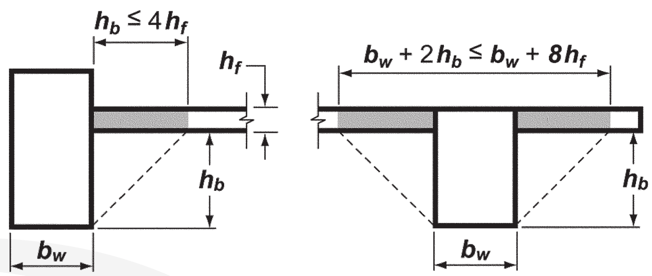

Fig. 3.2.1.3—Example of portion of slab to be included with beam.

$$\beta_t = \frac{E_{cb}C}{2E_{cs}I_s} \qquad (3.2.1.3a)$$

$$C = \sum\left(1 - 0.63\frac{x}{y}\right)\frac{x^3y}{3} \qquad (3.2.1.3b)$$

The constant C for T- or L-sections can be evaluated by dividing the section into separate rectangular parts, as defined in Fig. 3.2.1.3, and summing the values of C for each part.

The effect of the torsional stiffness parameter β_t is to assign all of the exterior negative factored moment to the column strip, and none to the middle strip, unless the beam torsional stiffness is high relative to the flexural stiffness of the supported slab. In the definition of β_t, the shear modulus has been taken as $E_{cb}/2$. Where walls are used as supports along column lines, they can be regarded as very stiff beams with a $\alpha_{f1}\ell_2/\ell_1$ value greater than 1. Where the exterior support consists of a wall perpendicular to the direction in which moments are being determined, β_t may be taken as zero if the wall is of masonry without torsional resistance, and β_t may be taken as 2.5 for a concrete wall with large torsional resistance that is monolithic with the slab.

For the purpose of establishing moments in the half column strip adjacent to an edge supported by a wall, ℓ_n in Eq. (3.1.3) may be assumed equal to ℓ_n of the parallel adjacent column-to-column span, and the wall may be considered as a beam having a moment of inertia, I_b ,equal to infinity. Where supports consist of columns or walls extending for a distance equal to or greater than $(3/4)\ell_2$ used to compute

Table 3.2.1.6—Distribution in percent of factored moment in beams

$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	85
$(\alpha_{f1}\ell_2/\ell_1) = 0$	0

Note: Linear interpolations can be made between values shown.

M_o, negative moments should be considered to be uniformly distributed across ℓ_2.

Column strips should be proportioned to resist the portions in percent of positive factored moments shown in Table 3.2.1.3c.

3.2.1.4 *Factored moments in middle strips*—That portion of the negative and positive factored moments not resisted by column strips should be proportionately assigned to corresponding half middle strips. Each middle strip should be proportioned to resist the sum of the moments assigned to its two half middle strips. A middle strip adjacent to and parallel with a wall-supported edge should be proportioned to resist twice the moment assigned to the half middle strip corresponding to the first row of interior supports.

3.2.1.5 *Modification of factored moments*—A modification of 10 percent in negative and positive factored moments, calculated in accordance with 3.2.2.2, is permitted provided the total static moment for a panel in the direction considered is not less than M_o required by Eq. (3.1.3). For example, a 10 percent reduction of negative factored moment should follow with a corresponding increase of positive factored moment. This is intended to recognize a limited amount of inelastic behavior and moment redistribution can occur in slabs that were analyzed with the DDM.

3.2.1.6 *Factored moments in beams*—Factored moments in beams between supports are proportioned in percent as shown in Table 3.2.1.6.

Loads assigned directly to beams are in addition to the uniform dead load of the slab; uniform superimposed dead loads, such as the ceiling, floor finish, or assumed equivalent partition loads; and uniform live loads. All of these loads are normally included with q_u in Eq. (3.1.3). Linear loads applied directly to beams include partition walls over or along beam centerlines and additional dead load of the projecting beam stem. Concentrated loads include posts above or hangers below the beams or induced force from seismic overturning. For the purpose of assigning directly applied loads, only loads located within the width of the beam stem should be considered as directly applied to the beams. The effective width of a beam as defined in Fig. 3.2.1.3 is solely for strength and relative stiffness calculations. Line loads and concentrated loads located on the slab away from the beam stem require special consideration to determine their apportionment to slab and beams. For slabs with beams between supports, the slab portion of column strips should be proportioned to resist that portion of the column strip moments not resisted by beams.

3.2.1.7 *Factored shear in slab systems with beams*—Resistance to total shear occurring on a panel based on computation of slab shear strength on the assumption that loads are distributed to supporting beams is permitted. Shear

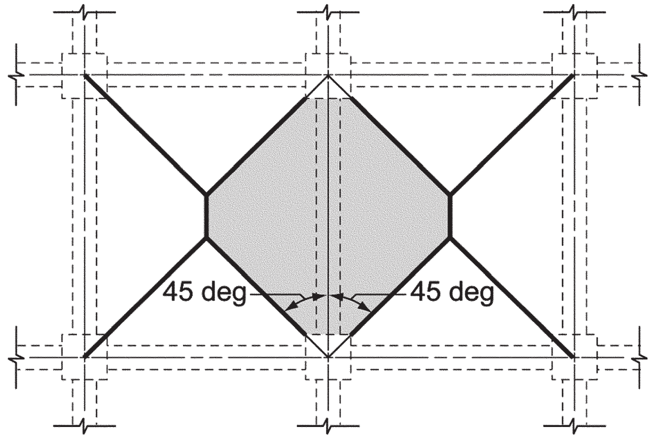

Fig. 3.2.1.7—Tributary area for shear on an interior beam.

strength for beams should satisfy ACI 318-14, Sections 9.5.3 and 22.5.

Beams with $\alpha_{f1}\ell_2/\ell_1$ equal to or greater than 1.0 are proportioned to resist shear caused by factored loads on tributary areas that are bounded by 45-degree lines drawn from the corners of the panels and the centerlines of the adjacent panels parallel to the long sides. The tributary area for computing shear on an interior beam is shown shaded in Fig. 3.2.1.7.

If the stiffness for the beam, $\alpha_{f1}\ell_2/\ell_1$, is less than 1.0, the shear on the beam can be obtained by linear interpolation, assuming beams resist no load at $\alpha_{f1} = 0$. In such cases, the beams framing into the column will not account for all of the shear force applied on the column. The remaining shear force will produce shear stresses in the slab around the column, which should be checked in the same manner as for flat slabs.

In addition to shears calculated from slab tributary areas, beams should be proportioned to resist shear caused by factored loads applied directly on beams. Note that the proportional shear in slab systems with beams described in this section do not apply to calculation of torsional moments on the beams. Torsional moments should be based on the calculated flexural moments acting on the sides of the beam.

3.2.1.8 *Factored moments in columns and walls*—Columns and walls built integrally with a slab system should resist moments caused by factored loads on the slab system. At an interior support, columns or walls above and below the slab should resist the factored moment specified by Eq. (3.2.1.8) in direct proportion to their stiffnesses unless a general analysis is made.

$$M_u = 0.07[(q_{Du} + 0.5q_{Lu})\ell_2\ell_n^2 - q'_{Du}\ell_2'(\ell_n')^2 \quad (3.2.1.8)$$

where q'_{Du}, ℓ_2', and ℓ_n' refer to the shorter span.

Equation (3.2.1.8) refers to two adjoining spans, with one span longer than the other, and with full dead load plus one-half live load applied on the longer span with only dead load applied on the shorter span. Design and detailing of the reinforcement transferring the moment from the slab to edge column is critical to both performance and safety of flat

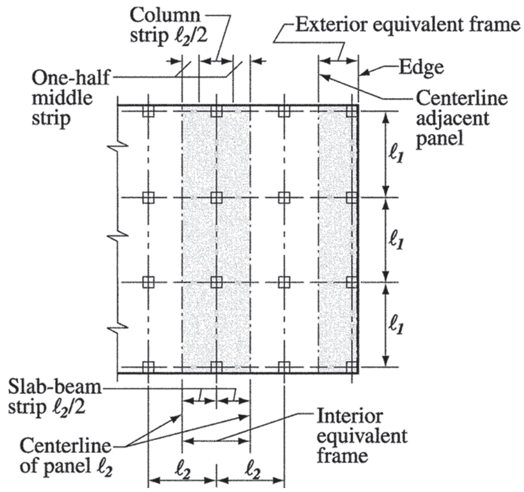

Fig. 3.2.2—Definitions of equivalent frame.

slabs or flat plates without edge beams, or cantilever slabs. Complete details are shown on construction documents, such as concentration of reinforcement over the column by closer spacing or additional reinforcement.

3.2.2 *Equivalent Frame Method (EFM)*—The EFM involves the representation of a three-dimensional slab system by a series of two-dimensional frames that are then analyzed for loads acting in plane of frames. The negative and positive moments so determined at the critical design sections of the frame are distributed to the slab sections in accordance with 3.2.1.3 (column strips), 3.2.1.4 (middle strips), and 3.2.1.6 (beams). The EFM is based on studies by Corley et al. (1961), Jirsa et al. (1963), and Corley and Jirsa (1970). Unlike the DDM, the EFM can be used for slab systems with irregular column layouts and pattern loading with or without lateral forces.

The EFM was developed when the primary structural analysis tool was the moment-distribution method. Specialized computer programs have been developed for this analysis, and give a way of using ordinary plane-frame structural analysis programs for this task (Schaeffer 1999).

Application of the equivalent frame to a regular structure is illustrated in Fig. 3.2.2. The three-dimensional building is divided into a series of two-dimensional frame bents (equivalent frames) oriented along column or support centerlines, with each frame extending the full height of the building. The width of each equivalent frame is bounded by the centerlines of the adjacent panels. The complete analysis of a slab system for a building consists of analyzing a series of equivalent (interior and exterior) frames spanning longitudinally and transversely through the building.

The equivalent frame comprises three parts:

1) The horizontal slab strip, including any beams spanning in the direction of the frame

2) The columns or other vertical supporting members, extending above and below the slab

3) The elements of the structure that provide moment transfer between the horizontal and vertical members.

Where metal column capitals are used, it is permitted to consider their contributions to stiffness and resistance to moment and shear. Change in column and slab length due to direct stress and deflections due to shear can be neglected.

3.2.2.1 *Analysis and design considerations*—All sections of slabs and supporting members analyzed by EFM should be proportioned for moments and shears obtained:

a) The structure is considered to be made up of equivalent frames on column lines taken longitudinally and transversely through the building

b) Each frame consists of a row of columns or supports and slab-beam strips, bounded laterally by the centerline of panel on each side of the centerline of columns or supports

c) Columns or supports are assumed to be attached to slab-beam strips by torsional members (3.2.2.4) transverse to the direction of the span for which moments are being determined and extending to bounding lateral panel centerlines on each side of a column

d) Frames adjacent and parallel to an edge are bounded by that edge and the centerlines of adjacent panels

e) Analysis of each equivalent frame in its entirety is permitted. Alternatively, for gravity loading, a separate analysis of each floor or roof with far ends of columns considered fixed is also permitted

f) Where slab-beams are analyzed separately, determination of the moment at a given support, assuming that the slab-beam is fixed at any support two panels distant therefrom, is permitted, provided the slab continues beyond the points of fixity.

The simplifications permitted under e) and f) were concessions to manual calculation procedures such as moment distribution.

3.2.2.2 *Slab-beams*—In a slab-beam system, a support is defined as a column, capital, bracket, or wall; a beam is not considered to be a support member for the equivalent frame. Determining the moment of inertia of slab-beams at any cross section outside of joints or column capitals using the gross area of concrete is permitted. Consider variation in moment of inertia along the axis of slab-beams. The moment of inertia of slab-beams from the center column to the face of the column, bracket, or capital is assumed equal to the moment of inertia of the slab-beam at the face of the column, bracket, or capital divided by the quantity $(1 - c_2/\ell_2)^2$, where c_2 and ℓ_2 are measured transverse to the direction of the span for which moments are being determined.

3.2.2.3 *Columns*—Column stiffness is based on column length from middepth of the slab above to middepth of the slab below. The column moment of inertia is computed on the basis of its cross section, taking into account the increase in stiffness provided by the capital, if any. Consider variation in moment of inertia along the axis of columns. Moment of inertia of columns from top to bottom of the slab-beam at a joint can be assumed to be infinite.

When slab-beams are analyzed separately for gravity loads, the concept of an equivalent column, combining the stiffness of the slab-beam and torsional member into a composite element, is used. The column flexibility is modified to account for the torsional flexibility of the slab-

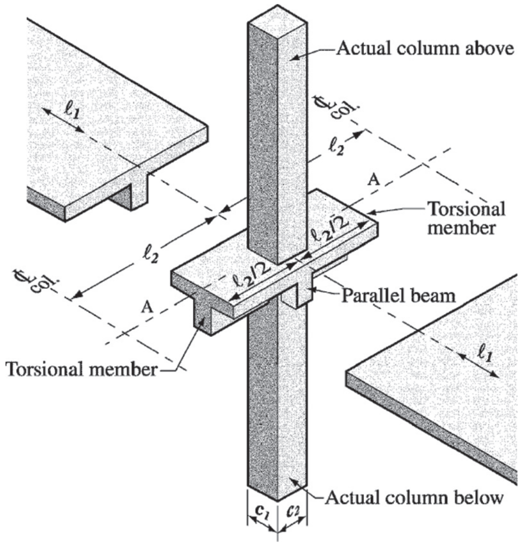

Fig. 3.2.2.3—Definitions of equivalent frame.

to-column connection that reduces its efficiency for transmission of moments. The equivalent column consists of the actual columns above and below the slab-beam, plus attached torsional members on each side of the columns extending to the centerline of the adjacent panels, as shown in Fig. 3.2.2.3.

3.2.2.4 *Torsional members*—Torsional members (3.2.2.1) should be assumed to have a constant cross section throughout their length consisting of the larger of a), b), and c):

a) A portion of slab having a width equal to that of the column, bracket, or capital in the direction of the span for which moments are being determined

b) For monolithic or fully composite construction, the portion of slab specified in a), plus that part of the transverse beam above and below the slab

c) The transverse beam as defined in Fig. 3.2.1.3.

Computation of the stiffness of the torsional member requires several simplifying assumptions. If no transverse-beam frames into the column, a portion of the slab equal to the width of the column or capital is assumed to be the torsional member. If a beam frames into the column, T-beam or L-beam action is assumed, with the flanges extending on each side of the beam a distance equal to the projection of the beam above or below the slab but not greater than four times the thickness of the slab. Additionally, it is assumed that no torsional rotation occurs in the beam over the width of the support.

The member sections used for calculating the torsional stiffness have been defined previously. An approximate expression for the stiffness of the torsional member, based on the results of three-dimensional analyses of various slab configurations (Corley et al. 1961; Jirsa et al. 1963; Corley and Jirsa 1970) is given in Eq. (3.2.2.4a).

$$K_t = \sum \frac{9E_{cs}C}{\ell_2\left(1-\dfrac{c_2}{\ell_2}\right)^3} \tag{3.2.2.4a}$$

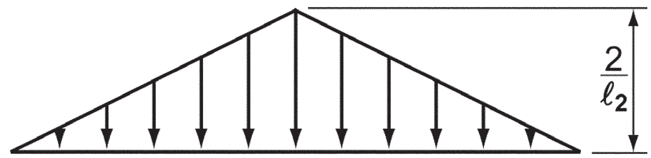

Fig. 3.2.2.4—Distribution of unit twisting moment along column centerline.

where $C = \sum\left(1-0.63\dfrac{x}{y}\right)\dfrac{x^3y}{3}$ (3.2.2.4b)

Studies of three-dimensional analyses of various slab configurations suggest that a reasonable value of the torsional stiffness can be obtained by assuming a moment distribution along the torsional member that varies linearly from a maximum at the center of the column to zero at the middle of the panel. The assumed distribution of unit twisting (or torsional) moment along the column centerline is shown in Fig. 3.2.2.4, in which the unit for $2/\ell_2$ is angle per unit length; and because the length is 12, the resultant is unity.

Where beams frame into columns in the direction of the span for which moments are being determined, the torsional stiffness should be multiplied by the ratio of the moment of inertia of the slab with such a beam to the moment of inertia of the slab without such a beam.

Stiffness of the equivalent frame based on the flexibility of the column and torsional member can be expressed by Eq. (3.2.2.4c) (Schaeffer 1999)

$$\frac{1}{K_{ec}} = \frac{1}{\Sigma K_c} + \frac{1}{K_t} \tag{3.2.2.4c}$$

3.2.2.5 *Arrangement of live load*—When the loading pattern is known, the equivalent frame is analyzed for that load. Except for the loading condition described in the next paragraph, maximum positive factored moment near midspan of a panel can be assumed to occur with three-fourths of the full factored live load on the panel and on alternate panels. The maximum negative factored moment in the slab at a support can be assumed to occur with three-fourths of the full factored live load on adjacent panels only. Factored moments should be taken not less than those occurring with full factored live load on all panels.

When the unfactored live load is variable but does not exceed three-fourths of the unfactored dead load, or the nature of live load is such that all panels will be loaded simultaneously, it is acceptable to assume that maximum factored moments occur at all sections with full factored live load on the entire slab system.

The use of only three-fourths of the full factored live load for maximum moment loading patterns is based on the fact that maximum negative and maximum positive live load moments cannot occur simultaneously and that redistribution of maximum moments is, therefore, possible before failure occurs. This procedure, in effect, permits some local overstress under the full factored live load when distributed in the prescribed manner, but still ensures that the ultimate strength of the slab system after redistribution of moment is not less than that required to resist the full factored dead and live loads on all panels.

3.2.2.6 *Factored moments*—At interior supports, the critical section for negative factored moment in both column and middle strips is at the face of rectilinear supports, but not farther away than $0.175\ell_1$ from the center of a column. At exterior supports with brackets or capitals, the critical section for negative factored moment in the span perpendicular to an edge is at a distance from the face of supporting element not greater than one-half the projection of bracket or capital beyond the face of the supporting element. Circular or regular polygon-shaped supports are treated as square supports with the same area for location of the critical section for negative moment, as illustrated in Fig. 3.1.3.

Using the centerline moment and shear from equivalent frame analysis, adjust the negative factored moments to the face of the supports. At interior supports, the adjustment is commonly taken as $V_u c_1/3$. At an exterior support, the adjustment is commonly taken as $V_u c_1/4$ to limit reductions in the exterior negative moment.

Where slab systems within limitations of 3.2.1.1 are analyzed by EFM, it is permitted to reduce the resulting computed moments in such proportion that the absolute sum of the positive and average negative moments need not exceed the value obtained from Eq. (3.1.3). This relaxation is based on long satisfactory experience with analyses when applicable limitations are met. Distribution of moments at critical sections across the slab-beam strip of each frame to column strips, middle strips and beam, and (as described in 3.2.1.3, 3.2.1.4, and 3.2.1.6) may be considered in conjunction with limitation f) of 3.2.1.1.

3.3—Finite element analysis

The finite element method is a tool for analyzing reinforced concrete slabs, particularly for complex slab systems. This method is widely used by practicing engineers and researchers. There are many commercial structural analysis software packages that have the finite element analysis option. Because this method is an approximate numerical method, it is essential to understand the relationship between the actual behavior of the structure and the numerical simulation. Results of the finite element model should be verified before being used for design applications. As a minimum, verification should include a check of summation of vertical and horizontal reactions and applied forces satisfying force equilibrium.

The finite element method is based on several assumptions, including:

1) Most commercial finite element packages use linear analysis, whereas reinforced concrete is a nonlinear material

2) The finite element method is based on isotropic and homogenous material, whereas reinforced concrete is a composite material

3) Derivations of the various plate elements used for slab modeling are based on small displacement and rotation assumptions. The term "small displacement" means, in general, small relative to the thickness of the slab; this assumption allows the use of linear analysis that ignores geometric nonlinearity.

In finite element modeling, selecting element type, mesh size, material properties, and boundary conditions are critical to perform sufficiently accurate simulation of reinforced concrete slabs. The mesh size or number of elements used in the model can significantly affect reliability of the results. In general, the finer the mesh or the higher the number of elements, the more reliable the results.

The type of finite element used in the analysis can significantly affect the results because the various elements used for slab analysis are derived from different assumptions. For example, Kirchoff plate bending is derived for thin plate applications and accounts for flexural deformation only, whereas Mindlin plate elements are derived for moderately thick plate applications and accounts for both flexural and shear deformations (Logan 2002). Thin plate refers to a plate with thickness that is much smaller than its in-plane dimensions, and thick plate refers to a plate with thickness that is greater than one-tenth the span of the plate. Finite element formulation is based on shape functions that interpolate the solution between the nodes. Elements with higher-order shape functions are more accurate than elements with lower-order shape functions, and elements with mid-nodes are more accurate than elements with edge nodes only.

Another important factor that may affect the finite element results is selection of boundary conditions. The types of boundary conditions used to model slabs pertain to support boundary conditions and to slab-to-column connections. There are two approaches to model boundary conditions: in the first approach, the columns supporting the slab and the columns above the slab are modeled using frame elements or three-dimensional solid elements. This approach accounts for the flexural effects of the columns as well as the columns above the slab. In the second approach, however, the column supports are modeled as pin or as fixed supports, or instead as springs with finite elastic stiffness to improve the behavior of the model. Gentry (1986) investigated these two approaches and concluded that using frame elements or three-dimensional elements are preferable to using pin or fixed supports to simulate columns. For modeling slab-supporting beams, the two approaches mentioned previously may be used. Using frame elements of three-dimensional elements is preferable to using a series of pin or fixed supports along the beam lines. Note that using pin or fixed supports does not accurately simulate the actual stiffness of the connection between the slab and the supporting beam or column. Also, another consideration in modeling the boundary conditions is the physical size of connection versus what is used in the model. When the supporting member is connected to a node, the connection occurs over an infinitesimal area, where the physical connection occurs over a defined area. This will result in an unrealistic stress concentration at the support, an inaccurate clear span that results in greater moment in the slab, and in an inaccurate moment at the face of the column.

Once the finite element analysis is performed and the solution converged, verify the results before using them

Fig. 3.4a—Rectangular slab panel (courtesy of Nawy [2011]).

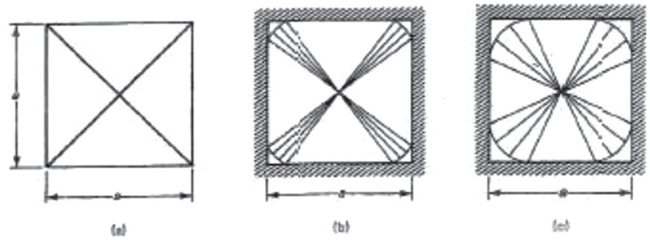

Fig. 3.4b—Idealized failure mechanism of a square panel with different boundary conditions (courtesy of Nawy [2011]).

for design purposes. Check the equilibrium of the structure to verify the sum of the reactions versus the applied load. Global displacement of the structure should be verified and confirmed as small in comparison to the structure's geometry. The engineer should critically investigate the resulting output from the model. This would include the deflected shape, and stress and displacement contour plots.

3.4—Yield-line theory

The yield-line theory is an analysis method that selects the applicable upper-bound model that gives the peak ultimate load capacity of structural slab or plate in a flexural mode of failure and perfect plasticity. Such a load level is occasionally termed as an "upper-bound" solution. A succession of hinge bands that develop at such a load level are idealized by lines, hence, the name "yield-line theory" (Johansen 1962). The theory assumes rigid-plastic behavior; that is, each concrete plate or slab segment stays planar up to collapse, producing rigid planar segments (Fig. 3.4(a) and 3.4(b)). Consequently, deflection is not considered in theory, nor are the compressive or tensile membrane forces that may act in the plane of the slab or plate being analyzed. The plates or slabs are considered significantly under-reinforced in this procedure and, therefore, require ductility. This is achieved by limiting the reinforcement ratio to 1.0 percent of the section needed for the controlling flexural yield-line moment and steel strain within a range of 0.010. This higher reinforcement percentage is necessitated by the dominance of shear-flexure failure mechanism at the column boundaries in the flat plates.

Slab thickness obtained by a yield-line analysis is often thinner than one from other lower-bound methods, such as DDM, EFM, or the strip method. Consequently, it is important to rigorously apply the serviceability requirements for deflection and crack control as recommended in ACI 224R and ACI 435R when determining the thickness of two-way slab or plate when this theory is used in analysis and design.

One distinct advantage of yield-line theory is the ability to provide rapid solutions for any shape of plate; most of the other approaches discussed are applicable only to rectangular shapes. Extending them to other shapes requires more rigorous computations for boundary effects. This theory enables an engineer to obtain the load capacity for a triangular, trapezoidal, rectangular, circular, or any other shape subjected to distributed or concentrated loads, provided that the failure mechanism is known or predictable. Because most failure patterns are presently identifiable, solutions are readily obtained (Nawy 2011; Park and Gamble 1999).

3.5—Strip method analysis

Other methods of analysis, such as the strip method, that conform to the fundamental principles of structural behavior of two-way action slabs and plates are also available, as discussed in 3.1.1. The strip method, in contrast to the yield-line method, is a lower-bound solution to the collapse load, where twisting moments are considered absent from the analysis as the plate is segmented into beam strips. The solution is, therefore, based on beam action only through satisfying equilibrium at all points in the slab. Consequently, it is applicable for proportioning rectangular two-way slabs and plates as compared to the yield-line method's versatility in its application to any conceivable shape, as discussed in 3.4. The strip method allows some freedom to the designer for choosing how the slab should resist load, as well as for analyzing and designing the slab to meet the loading conditions (Schaeffer 1999).

CHAPTER 4—DESIGN PROCEDURES

4.1—General

As stated in 3.1.1, two-way slab design is based directly on the fundamental principles of structural mechanics, provided it can be demonstrated explicitly that all safety and serviceability criteria are satisfied. For a slab system supported by columns or walls, the effective support area defined by the intersection of the bottom surface of the slab or drop panel (if there is one), is based on the dimensions c_1 and c_2 and the clear span ℓ_n. It includes the largest right circular cone, right pyramid, or tapered wedge with surfaces located within the column and capital or bracket, and oriented no greater than 45 degrees to the axis of the column. Circular or regular polygon-shaped supports should be treated as square supports with the same area as shown in Fig. 3.1.3. The design of two-way slabs considers the following general provisions:

a) A column strip is a design strip with a width on each side of a column centerline equal to $0.25\ell_1$ or $0.25\ell_2$, whichever is less

b) The column strip includes beams, if any

c) The middle strip is a design strip bounded by two column strips

d) A panel is bounded by column, beam, or wall centerlines on all sides

e) For monolithic or fully composite construction, a beam includes that portion of slab on each side of the beam web extending a distance equal to the projection of the beam above or below the slab, whichever is greater, but not greater than four times the slab thickness.

4.2—Gravity loading

For gravity loads, analysis of a slab system should consider the aspect ratio of each slab panel and the relative stiffness of the slab panels, supporting beams (if any), and supporting columns or walls. Gravity dead load includes self-weight plus long-term superimposed loads, such as flooring, ceiling, suspended nonstructural elements, or floor-mounted equipment. Live load is based on building occupancy and is prescribed in the governing building code. Permanent partitions can be included as either dead load or additional live load based on the type of occupancy, or as prescribed in the governing building code.

For light-frame construction supported on a podium flat plate or flat slab, engineering judgment should be exercised in specifying superimposed gravity dead and live loads resulting from bearing walls. Line load perpendicular to the direction of span should be included as a concentrated load in design. Line load parallel to the direction of the span should be included as a distributed load over an effective slab width not to exceed four times the thickness of slab.

4.3—Flexural design

As stated in 3.1.1, the slab design can be achieved through the combined use of classic solutions based on a linearly elastic continuum and evaluation of the stress conditions around the supports in relation to shear and torsion as well as flexure. The designer should recognize the limitations on the applicability of simplified design assumptions. Selection of physical dimensions of the slab should be compared to common office practice on the basis of knowledge of the expected loads and the reliability of the calculated stresses and deformations of the structure. Apply appropriate load factors from ACI 318-14, Section 5.2, unless other requirements are given in the governing building code.

Engineers note that current specified load and ϕ-factors lead to tensile reinforcement flexure stresses approximately 10 percent higher than codes prior to 2002. The increase reinforcement would lead to a decrease of the crack widths, but could simultaneously increase the number of cracks of narrower width, which is the preferable end result. This is especially of concern at an interior column of a flat plate, where the local reinforcement stresses were already high due to stress concentrations of flexural, shear, and torsion that have traditionally been ignored. Flexural reinforcement in column strips should be proportioned on the basis of the full negative moment value without reduced redistribution of negative moments permitted in ACI 318-14, Section 6.6.5.1, as an adequate safety margin.

4.4—Two-way action slab shear

The design of two-way slab shear is described in subsequent sections of this guide. For slabs of uniform thickness, it is sufficient to check two-way slab shear at one critical section. When shear controls at the support, provide stirrups or stud reinforcement. For slabs with changes in thickness, such as the edge of drop panels or shear caps, check shear at several critical sections. For edge columns at points where the slab cantilevers beyond the column, the critical perimeter will either be three- or four-sided.

4.5—Critical section

The critical section for shear in slabs subjected to bending in two directions follows the perimeter close to the edge of the effective support area. The shear stress acting on this section at factored loads is a function of $\sqrt{f_c'}$ and the ratio of the side dimension of the column to the effective slab depth. A much simpler design equation is derived by assuming a pseudo-critical section located at a distance $d/2$ from the periphery of the effective support area. When this is done, the shear strength is almost independent of the ratio of column size to slab depth. For rectangular columns, this critical section is defined by straight lines drawn parallel to and at a distance $d/2$ from the column edges. ACI 318-14, Section 22.6.4.1.1, allows the use of a rectangular critical section.

4.6—Openings in slab systems

In general, openings of any size are permitted in slab systems if it can be shown by analysis that the design strength is at least equal to the required strength set forth in ACI 318-14, Sections 8.4 and 8.5, and that all serviceability conditions, including the limits on deflections, are met. The locations of the effective portions of the critical section near typical openings and free edges are discussed further in 4.6.2 of this guide (Joint ACI-ASCE Committee 426 1974).

4.6.1 *Permissible openings without analysis*—As an alternate to analysis, openings are permitted in slab systems without beams if in accordance with the following conditions:

a) Openings of any size are permitted in the area common to the intersecting middle strips, provided the total amount of reinforcement required for the panel without the opening is maintained;

b) In the area common to the intersecting column strips, not more than one-eighth the width of column strip in either span should be interrupted by openings. An amount of reinforcement equivalent to that interrupted by an opening should be added on the sides of the opening;

c) In the area common to one column strip and one middle strip, not more than one-fourth of the reinforcement in either strip may be interrupted by openings. An amount of reinforcement equivalent to that interrupted by an opening should be added on the sides of the opening;

d) Shear guidelines at a critical section described in 4.5 reduced by the effects of openings should be followed.

4.6.2 *Permissible openings with analysis*—When openings in slabs are located at a distance less than 10 times the slab thickness from a concentrated load or reaction area, or when openings in flat slabs are located within column strips, the critical slab sections for shear should be modified as follows:

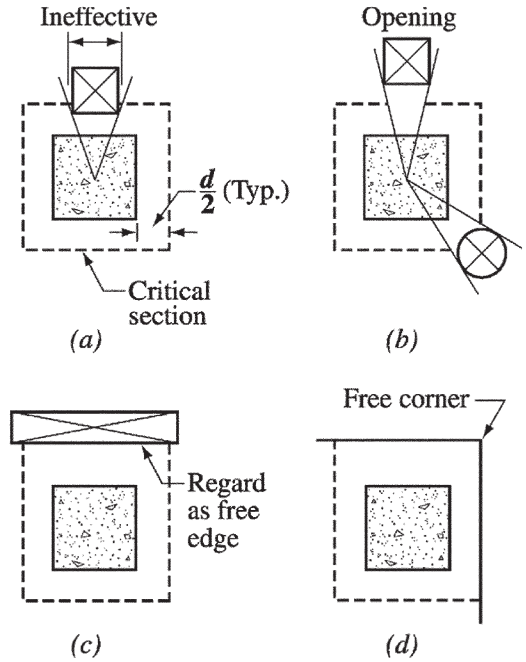

Fig. 4.6.2—Effect of openings and free edges. (Note: effective perimeter shown with dashed lines.)

a) For slabs without shearheads, that part of the perimeter of the critical section that is enclosed by straight lines projecting from the centroid of the column, concentrated load, or reaction area and tangent to the boundaries of the openings should be considered ineffective;

b) For slabs with shearheads, the ineffective portion of the perimeter should be one-half of that defined in 4.6.1, condition a).

The location of the effective portions of the critical section near typical openings and free edges is shown by the dashed line in Fig. 4.6.2.

4.7—Unbalanced moments

For slab systems without beams, tests (Hanson and Hanson 1968) and experience have shown that measures should be taken to resist the torsional and shear stresses. The fraction of unbalanced moment transferred by eccentricity of shear at slab-column connections is defined as γ_v and is equal to ($1 - \gamma_f$). The calculated shear stresses in the slab critical section should conform to the requirements of ACI 318-14, Section 8.4.4.2 (refer to design examples in ACI 421.1R).

A fraction of the unbalanced moment given by $\gamma_f M_{sc}$ should be considered to be transferred by flexure, where M_{sc} is the moment to be transferred from slab to column. All reinforcement resisting that part of the moment to be transferred to column by flexure should be placed within an effective slab width between lines that are one and one-half slab or drop panel thicknesses ($1.5h$) outside opposite faces of the column or capital, except $1.5h$ of the slab thickness should be used where shear cap is used. For a shear critical section in the shape of a closed rectangle of sides b_1 and b_2, ACI 318-14, Section 8.4.2.3.2 gives

$$\gamma_f = \frac{1}{1+(2/3)\sqrt{b_1/b_2}} \tag{4.7a}$$

and requires that the remainder of the unbalanced moment, given by $\gamma_v M_u$, should be considered transferred by eccentricity of shear about the centroid of the shear critical section, where

$$\gamma_v = 1 - \gamma_f \tag{4.7b}$$

ACI 421.1R gives equations for γ_v for shear critical sections of any shape and equations for planar shear stress distribution satisfying ACI 318 for a nonrectangular critical section of general shape. Some flexibility in distribution of unbalanced moments transferred by shear and flexure at both exterior and interior supports is possible. Interior, exterior, and corner supports refer to slab-column connections for which the critical perimeter for rectangular columns has four, three, or two sides, respectively. The shear critical section at $d/2$ from the outer peripheral lines of shear reinforcement generally follows a nonrectangular shape.

At exterior supports, for unbalanced moments about an axis parallel to the edge, the portion of moment transferred by eccentricity of shear, $\gamma_v M_u$, may be reduced, provided that the factored shear at the support (excluding the shear produced by moment transfer) does not exceed 75 percent of the design shear strength ϕV_c as defined in ACI 318-14, Section 22.6.5.1, for edge columns or 50 percent for corner columns. This reduction of γ_v and the corresponding increase in γ_f (Eq. (4.7b)) is permitted by ACI 318, but is not recommended in this guide; the analyses and the experimental data in these references do not justify the reduction in γ_v (Gayed and Ghali 2008; Ritchie et al. 2006).

In slab-column connections, a large degree of ductility is required because the interaction between shear and unbalanced moment is critical. When the factored shear is large, the column-slab joint cannot always mobilize all of the reinforcement provided in the effective width. The reduction of γ_v and the increase of γ_f for edge, corner, or interior slab-column connections are permitted only if the strain in the flexural reinforcement within the effective width develops a minimum net tensile strain of 0.010.

When a reversal of moments occurs at opposite faces of an interior support, both top and bottom reinforcement should be concentrated within the effective width. A ratio of top to bottom reinforcement of approximately two is considered a good practice.

4.8—Shear strength

For nonprestressed slabs, V_c is calculated by the smallest of the following

$$V_c = \left(2 + \frac{4}{\beta}\right)\lambda\sqrt{f_c'}b_o d \quad \text{(in.-lb)}$$
$$V_c = \left(2 + \frac{4}{\beta}\right)\lambda\sqrt{f_c'}b_o d/12 \quad \text{(SI)} \tag{4.8a}$$

$$V_c = \left(\frac{\alpha_s d}{b_o} + 2\right)\lambda\sqrt{f_c'}b_o d \quad \text{(in.-lb)}$$
$$V_c = \left(\frac{\alpha_s d}{b_o} + 2\right)\lambda\sqrt{f_c'}b_o d/12 \quad \text{(SI)} \tag{4.8b}$$

$$V_c = 4\lambda\sqrt{f_c'}b_o d \quad \text{(in.-lb)}$$
$$V_c = \lambda\sqrt{f_c'}b_o d/3 \quad \text{(SI)} \tag{4.8c}$$

where α_s is 40 for interior columns, 30 for edge columns, and 20 for corner columns (interior, edge, and corner columns refer to critical sections with four, three, and two sides, respectively).

For square columns, the shear stress due to factored loads in slabs subjected to bending in two directions is limited to value given in Eq. (4.8c). However, tests (Joint ACI-ASCE Committee 426 1974) have indicated that the value of $4\lambda\sqrt{f_c'}$ is not conservative when the ratio β of the lengths of the long and short sides of a rectangular column or loaded area is larger than 2.0. In such cases, the actual shear stress on the critical section at punching shear failure varies from a maximum of approximately $4\lambda\sqrt{f_c'}$ around the corners of the column or loaded area, down to $2\lambda\sqrt{f_c'}$ or less along the long sides between the two end sections. Other tests (Vanderbilt 1972) indicated that V_c decreases as the ratio b_o/d increases. Equations (4.8a) and (4.8b) were developed to account for these two effects.

For shapes other than rectangular, β is taken to be the ratio of the longest overall dimension of the effective loaded area to the largest overall perpendicular dimension of the effective loaded area, as illustrated for an L-shaped reaction area in Fig. 4.8. The effective loaded area is that area totally enclosing the actual loaded area, for which the perimeter is a minimum.

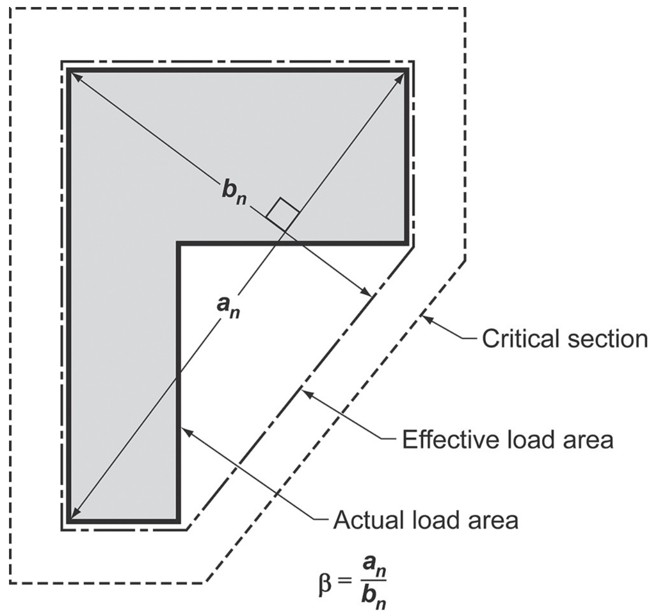

Fig. 4.8—Value of β for nonrectangular loaded area.

4.9—Post-tensioned slabs

Typical nonprestressed flat-plates, flat-slabs, or flat-plates with shear cap can be designed using ACI 318-14, Section 8.2.5. However, for the design of post-tensioned flat plates or flat slabs with unbonded tendons, there are additional items to consider. Most two-way post-tensioned slabs in the United States use unbonded tendons. Average effective prestress compressive stress levels are limited to a minimum 125 psi (0.86 MPa), per ACI 318-14, Section 8.6.2.1.

The moment due to unbalanced load, usually as a result of live load or unequal spans, can be distributed to each of the column-strip and middle-strip locations according to ACI 318-14, Section 8.4. The average compressive stress and the unbalanced moment should be checked at each critical section to ensure they are less than the prescribed stresses in ACI 318-14, Section 22.4.2.3. Provide the minimum temperature reinforcement or nonprestressed reinforcement required to resist tensile stress as required in ACI 318-14, Section 8.6.2. Use nonprestressed reinforcement and unbonded post-tensioning to find the nominal flexural strength of the critical sections of the column strip and middle strip. Required flexural strength shall be the sum of factored load moments and secondary moments using a load factor of 1.

At columns of two-way prestressed, shear strength is given by

$$V_c = (\beta_p\lambda\sqrt{f_c'} + 0.3f_{pc})b_o d + V_p \quad \text{(in.-lb)}$$
$$V_c = (\beta_p\lambda\sqrt{f_c'}/12 + 0.3f_{pc})b_o d + V_p \quad \text{(SI)} \tag{4.9}$$

where β_p is the smaller of 3.5 and $(\alpha_s d/b_o + 1.5)$. The term V_c may be computed by Eq. (4.9) if the following conditions are met; otherwise, 4.8 applies:

a) No portion of the column cross section should be closer to a discontinuous edge than four times the slab thickness

b) The value of $\sqrt{f_c'}$ used in Eq. (4.9) should not be taken greater than 70 psi (5.8 MPa)

c) In each direction, f_{pc} should not be less than 125 psi (0.86 MPa) nor be taken greater than 500 psi (3.45 MPa).

CHAPTER 5—SLAB REINFORCEMENT AND DETAILING

5.1—General

The area of reinforcement in each direction for two-way slab systems is determined from factored moments at critical sections. The requirements for flexural reinforcement are intended to provide nominal moment strength for slab

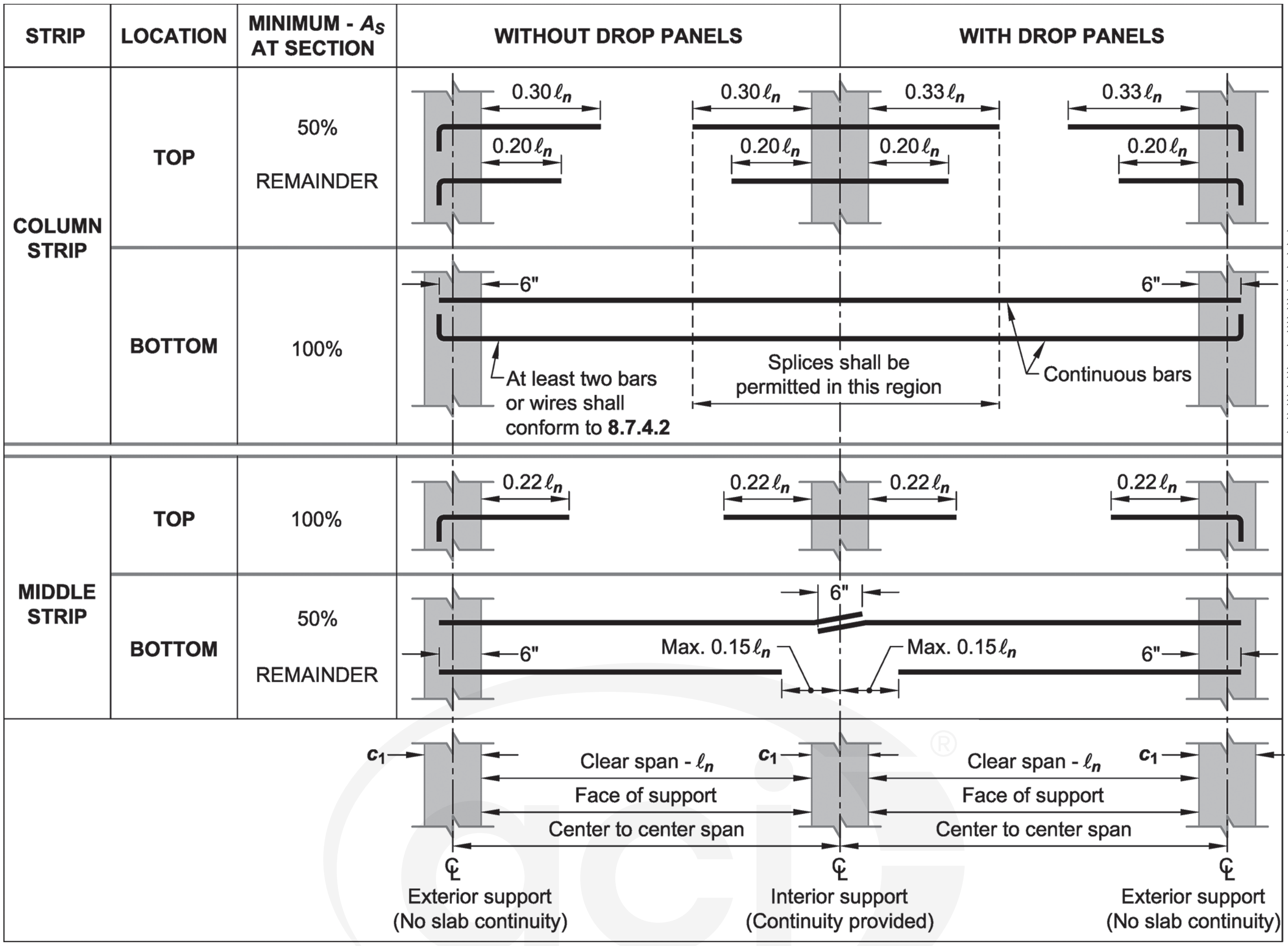

Fig. 5.2—Minimum extensions for deformed reinforcement in two-way slabs without beams (Fig. 8.7.4.1.3a, ACI 318-14).

action, reduce cracking, and provide for the possibility of loads concentrated on small areas of the slab.

5.1.1 *Minimum ratio of flexural reinforcement*—The minimum ratio of flexural reinforcement should be no less than the minimum shrinkage and temperature reinforcement and provided and distributed at the tension side of slabs in each direction. The ratio of area of reinforcement to gross area, b_h, of slab should not be less than the following:

(a) Where Grade 40 (280 MPa) or 50 (340 MPa) deformed bars are used: 0.0020

(b) Where Grade 60 (420 MPa) deformed bars or welded wire reinforcement are used: 0.0018

(c) Where reinforcement with f_y > 60,000 psi (420 MPa) (at 0.35 percent strain) are used: 0.0018 × 60,000/f_y (0.0018 × 420/f_y).

5.1.2 *Maximum reinforcement spacing*—Spacing of reinforcement required for flexure in solid slabs should not exceed 2*h*. Cracks start to generate at approximately 33 percent of the factored load and are generally wide, contributing to the reduction of the E_cI_g stiffness of the two-way slab or plate, and thereby leading to excessive deflections both short and long term. Serviceability control of cracking in two-way slabs and plates is best achieved when cracks follow the spacings of the orthogonal reinforcement rather than wide flexural crack patterns at early loading stages. A limit of 12 in. (300 mm) is recommended for maximum spacing of reinforcement in both orthogonal directions, as demonstrated in more than 100 large-scale two-way slab tests to failure (ACI 224R; Nawy 2001). The principle of using smaller-diameter bars at closer spacing rather than large-diameter bars at greater spacing tend to optimize the use of flexural reinforcement without the need for exceeding the total area of reinforcement required in design. At the same time, this helps to control cracking.

5.2—Slabs without beams

Minimum extension of reinforcement in slabs without beams is shown in Fig. 5.2. Where adjacent spans are unequal, extensions of negative moment reinforcement beyond the face of support should be based on requirements of the longer span.

5.3—Corner reinforcement

Unrestrained corners of two-way slabs tend to lift when loaded. If this lifting tendency is restrained by edge walls or beams, bending moments result in the slab. Provide reinforcement at the top and bottom exterior corners when α_f is greater than 1.0 to resist these moments and control cracking.

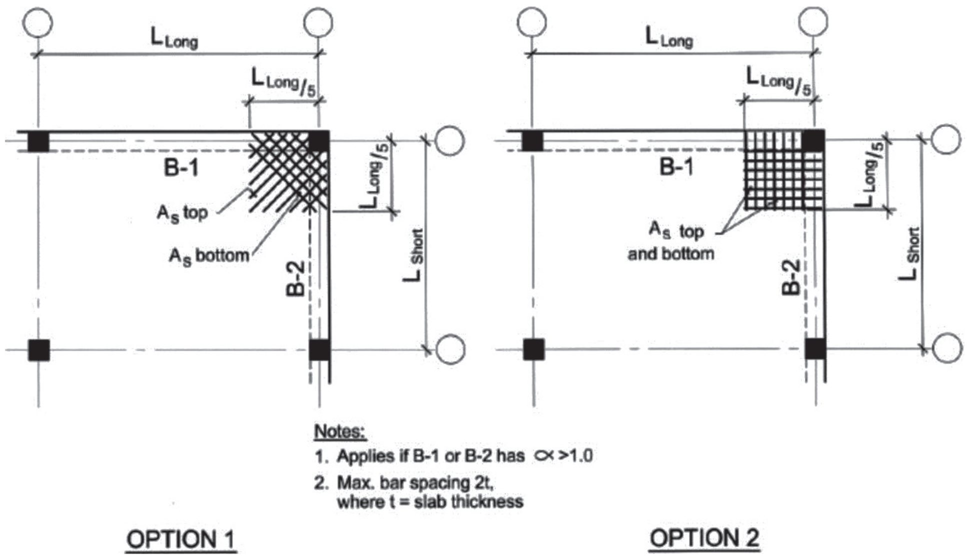

Fig. 5.3—Detail of corner reinforcement.

This reinforcement should be sufficient to resist a moment-per-unit width equal to the maximum positive moment-per-unit width in the slab at the exterior corners. The moment in the top of the slab may be assumed to act about an axis perpendicular to the diagonal from the corner. The moment in the bottom of the slab may be assumed to act about an axis parallel to the diagonal from the corner.

The corner reinforcement should be provided for a distance in each direction from the corner equal to one-fifth the longer span. The special reinforcement may be placed in a band parallel to the diagonal in the top of the slab and a band perpendicular to the diagonal in the bottom of the slab. Alternatively, reinforcement provided for flexure in the primary directions can be used if the special reinforcement is placed in two layers parallel to the sides of the slab in both the top and bottom of the slab (Fig. 5.3).

5.4—Slab with drop panel

When a drop panel is used to reduce the amount of negative moment reinforcement over the column of a flat slab, the thickness of the drop panel below the slab cannot be assumed to be greater than one-fourth the distance from the edge of the drop panel to the face of the column or column capital in computing the required slab reinforcement. When the drop panel extends less than one-sixth the span length from center-to-center of supports in each direction, the projection may be used as a shear cap to increase the shear strength of the slab but not the flexural strength. ACI 421.1R-08, Section 4.3.4, warns against specifying a small increase in slab thickness over a small area surrounding the column as a means to increase the critical perimeter and to reduce the shear stress under the required factored shear loads. For such conditions, potential shear cracks away from the shear cap and unbalanced moment should be investigated in detail.

5.5—Column strip reinforcement

All bottom bars or wires within the column strip in each direction should be continuous or spliced with Class B tension splices or with mechanical or welded splices satisfying ACI 318-14, Section 8.7.4.2.1. In general, splices should be located as shown in Fig. 5.2. At least two of the column strip bottom bars or tendons in each direction are required to pass within the column core and be anchored at exterior supports. The two continuous column strip bottom bars or tendons through the column are called integrity steel, and are provided to give the slab some residual strength following a single punching shear failure (Mitchell and Cook 1984).

Although ACI 318-14, Section 8.7.4.2 does not include an equation for the design of such integrity steel, reference may be made to CSA A23.3-04. The provision requires calculation of the area of steel passing through the column core and is given by Eq. (5.5)

$$\sum A_{sb} = 2V_{se}/f_y \tag{5.5}$$

When there is insufficient clearance for bottom bars to pass under the shearhead and through the column, such as in lift slab construction, bottom bars should pass through holes in the shearhead arms or within the perimeter of the lift collar.

5.6—Middle strip reinforcement

Middle strip bottom reinforcement may be stopped at a distance of $0.15\ell_n$ from the centerline of the interior support. Unless otherwise noted on construction document, middle strip bottom reinforcement should have a minimum extension of 6 in. (150 mm) from the face of the exterior support.

5.7—Bent bars

Bent bars were common in the earlier days of flat slabs to serve as surrogate shear reinforcement. In 1989, the provision for bent bars was removed from ACI 318-14, Section R8.7.4.1.3; however, bent bars are permitted when the depth-span ratio permits the use of bends of 45 degrees or

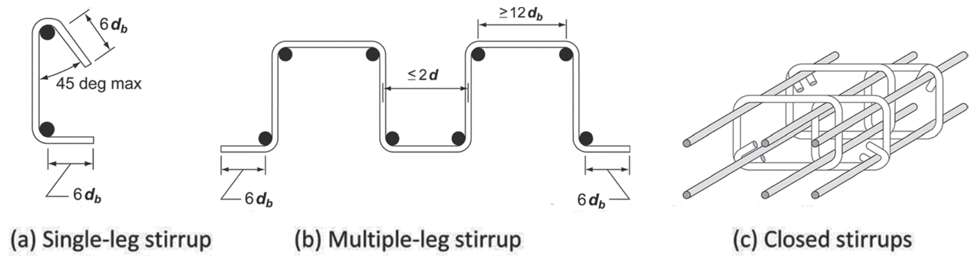

Fig. 5.8.1a—Single, multi-leg, or closed stirrup slab shear reinforcement.

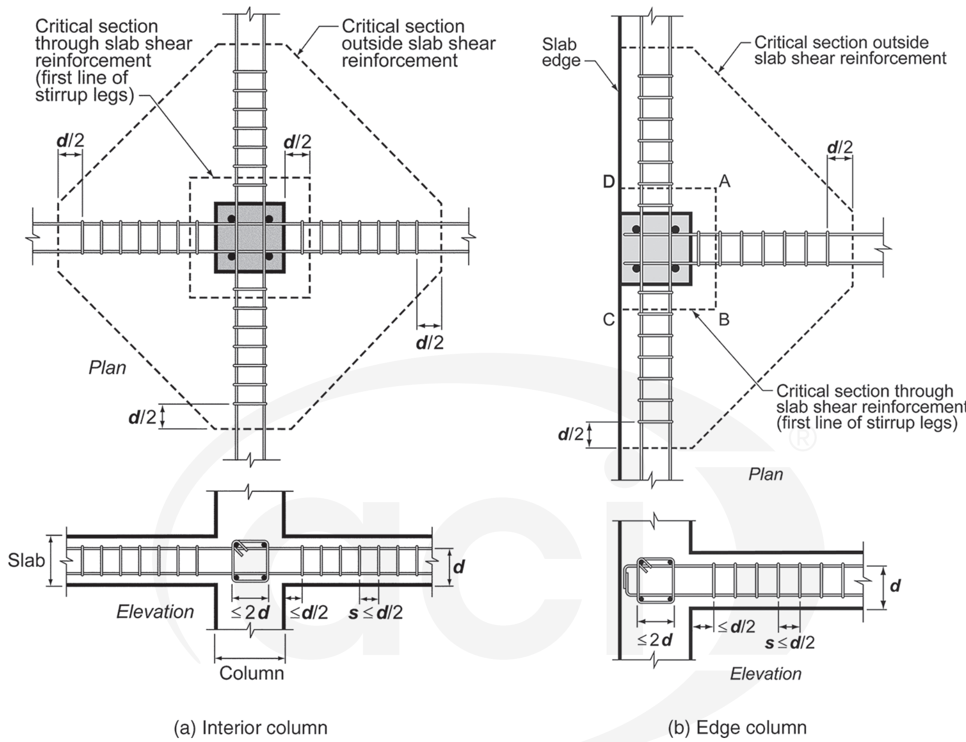

Fig. 5.8.1b—Arrangement of stirrup shear reinforcement.

less. In North America, this form of shear reinforcement in two-way slabs is rarely used.

5.8—Slab shear reinforcement

5.8.1 *Single or multiple leg stirrup type*—ACI 318-14, Section R8.7.6 sets out the principles of design for slab shear reinforcement and makes specific reference to stirrups, headed studs, and shearheads. Shear reinforcement consisting of properly anchored bars, wires and single- or multiple-leg stirrups, or closed stirrups can increase the punching shear resistance of slabs (Hawkins 1974). It is essential that shear reinforcement engage longitudinal reinforcement at both the top and bottom of the slab, as shown for typical details in Fig. 5.8.1a(a), (b), and (c). The minimum slab effective design depth d should not be less than 6 in. (300 mm) or 16 times the shear reinforcement diameter when such shear reinforcement is used. Anchorage of shear reinforcement, according to the requirements of ACI 318-14, Section 8.7.6.2, is difficult in slabs thinner than 10 in. (250 mm).

Shear reinforcement should be symmetrical about the centroid of the critical section when the unbalanced moment transfer is negligible for a slab-column connection. In accordance with ACI 318-14, Section 8.7.6.3, spacing limits for interior columns are illustrated in Fig. 5.8.1b(a). At edge columns or for interior connections where moment transfer is significant, closed stirrups are recommended in a pattern as symmetrical as possible. Although the average shear stresses on faces *AD* and *BC* of the exterior column in Fig. 5.8.1b(b) are lower than on face *AB*, the closed stirrups extending from faces *AD* and *BC* provide some torsional strength along the edge of the slab. The spacing limits shown correspond to slab shear reinforcement details that have been shown effective.

5.8.2 *Headed shear stud reinforcement*—Tests have established that punching shear in slabs can be effectively resisted by reinforcement consisting of vertical rods mechanically anchored at the top and bottom of slabs (ACI 421.1R). All types of mechanically anchored shear reinforcement are referred to as shear studs or studs. To be fully effective, the anchorage should be capable of developing the specified yield strength of the studs. The mechanical anchorage can be obtained by heads or strips connected to the studs

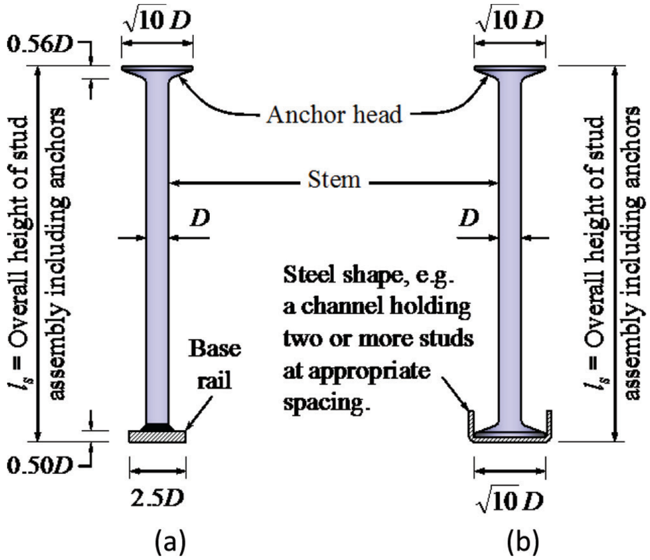

Fig. 5.8.2a—Stud assemblies: (a) single-headed studs welded to a base rail; and (b) double-headed studs crimped into a steel channel.

by welding. The heads can also be formed by forging the stud ends. Stud assemblies consisting of either a single-head stud attached to a steel base rail by welding is shown in Fig. 5.8.2a(a); a double-headed stud mechanically crimped into a nonstructural steel channel is shown in Fig. 5.8.2a(b). Mechanical properties of studs are specified in ASTM A1044/A1044M. Figure 5.8.2b is a top view of a slab that shows a typical arrangement of shear reinforcement (stirrup legs or studs) in the vicinity of an interior column. The shear reinforcement should be arranged on straight lines perpendicular to the column faces at the column corners. ACI 318 requires that the spacing g measured parallel to the column faces between corner lines shall not exceed $2d$. For optimum effectiveness, the overall height of the studs should be as great as possible. Ideally, the heads or the rail should have the minimum cover required for protection.

For full effectiveness, the head area of the studs has to be at least 10 times the area of the stem. This permits the design of headed stud shear reinforcement to be based on the specified yield strength f_{yt}. For headed stud shear reinforcement, ACI 318-14, Section 22.6.6.1 requires:

$$V_c = 3\lambda\sqrt{f_c'}b_o d \text{ and } V_n \le 8\lambda\sqrt{f_c'}b_o d \qquad \text{(in.-lb)}$$

$$V_c = \lambda\sqrt{f_c'}b_o d/4 \text{ and } V_n \le 2\lambda\sqrt{f_c'}b_o d/3 \qquad \text{(SI)}$$

When $v_u > 6\phi\sqrt{f_c'}$, $s \le d/2$ ($v_u > 0.5\phi\sqrt{f_c'}$)
When $v_u \le 6\phi\sqrt{f_c'}$, $s \le 0.75d$ ($v_u \le 0.5\phi\sqrt{f_c'}$)
For stirrups, ACI 318-14, Section 22.6.6.1 requires:

$$V_c = 2\lambda\sqrt{f_c'}b_o d \text{ and } V_n \le 6\lambda\sqrt{f_c'}b_o d \qquad \text{(in.-lb)}$$

$$V_c = 0.17\lambda\sqrt{f_c'}b_o d \text{ and } V_n \le 0.5\lambda\sqrt{f_c'}b_o d \qquad \text{(SI)}$$

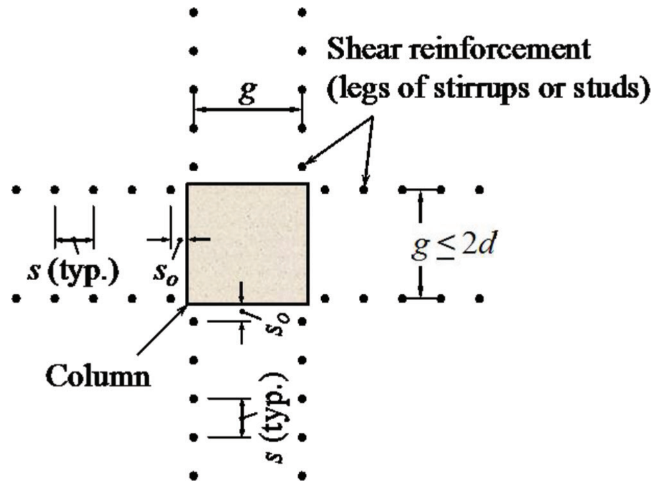

Fig. 5.8.2b—Top view of flat plate showing shear reinforcement arrangement in vicinity of interior column.

$s \le d/2$.

Refer to ACI 421.1R for detailed information on the application of headed shear studs.

5.9—Post-tensioned slabs

Until the mid-1970s, the placement of unbonded post-tensioning strands in flat plates and flat slabs was done using the patterns of the placement of reinforcement in reinforced concrete flat plates and flat slabs. This technique was labor intensive. Banded placement of strands developed as a result, which was much more cost-effective. In one direction, almost all the strands are banded within a small width (approximately $L/6$ distance) and are placed with a given drape. In the opposite direction, the strands are placed with uniform spacing, usually placed above the banded strands. The entire placement of strands looks more like a one-way slab in one direction and a wide beam, which are the banded strands, in the other direction. This placement results in equivalent strength as the previous placement. Banded placement of strands is the predominant practice in the United States today. The use of banded slabs also enhances the use of slab-column frames, as described in Chapter 7. It is important that at least two tendons pass through the column cores in the uniformly distributed tendon direction, and that these are anchored as near the slab edge as possible. These two tendons passing the column cores also serve as integrity reinforcement to meet the requirements of ACI 318-14, Section 8.7.5.6.

5.10—Bonded reinforcement in post-tensioned slabs

For two-way post-tensioned flat slab systems, the requirements for minimum area and distribution of bonded reinforcement in the positive and negative moment areas over supporting columns are described as follows.

In the positive moment areas where the extreme fiber stress in tension in the precompressed tensile zone at service loads does not exceed $2\sqrt{f_c'}$ after allowance for all prestressed losses, bonded reinforcement is not required.

Table 6.1.1—Minimum thickness of slabs

f_y, psi (MPa)	Without drop panels			With drop panels		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams		Without edge beams	With edge beams	
40,000 (280)	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
60,000 (420)	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
75,000 (520)	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

Note: For f_y between the values given, minimum thickness should be determined by linear interpolation. For slabs with beams along exterior edges, the value of α_f for the edge beam should not be less than 0.8.

When the computed tensile stress in concrete at service load exceeds $2\sqrt{f_c'}$, the minimum area of bonded reinforcement that should be provided is

$$A_s = \frac{N_c}{0.5f_y} \tag{5.10a}$$

where the value of f_y should not exceed 60,000 psi (420 MPa). Uniformly distribute bonded reinforcement across the precompressed tensile zone. Length of bonded reinforcement should be one-third the clear span length ℓ_n and centered in the positive moment area.

In the negative moment area over column supports, the minimum area of bonded reinforcement, A_s, required in the top of the slab in each direction is

$$A_s = 0.00075A_{cf} \tag{5.10b}$$

Distribute bonded reinforcement between lines that are 1.5h outside opposite faces of the column support. Provide at least four bars or wires in each direction. Reinforcement spacing should not exceed 12 in. (300 mm). Bonded reinforcement should extend one-sixth of the clear span ℓ_n on each side of support.

CHAPTER 6—SERVICEABILITY CONSIDERATIONS

6.1—Minimum slab thickness

Serviceability issues should be considered in the design of two-way flat slabs. These include determination of minimum slab thickness and immediate and long-term deflections. Long-to-short span ratio for two-way slabs is limited to 2-to-1. For slabs using nonprestressed reinforcement, the minimum slab thickness should conform to the prescriptive guideline based on 6.1.1 and 6.1.2. For deviation from the prescriptive guideline, slab deflection should be investigated based on 6.2.

6.1.1 *Slabs without interior beams spanning between supports*—For slabs without interior beams spanning between supports, the recommended minimum slab thicknesses are given in Table 6.1.1 as a function of ℓ_n, which is the length of clear span in the long direction measured face-to-face of supports in slabs without beams, and face-to-face of beams or other supports in other cases. For slabs without drop panels, minimum thickness should be at least 5 in. (125 mm). For slabs with drop panels, minimum thickness should be at least 4 in. (100 mm).

The minimum thicknesses described above have been developed through engineering and observational experience. Slabs conforming to these limits have not resulted in systematic problems related to stiffness for short- and long-term loads. Consider range of loads, material types and properties, boundary conditions, and environmental effects in determining the slab thickness through computational analysis as prescribed in ACI 435R-95. Slab deflection due to sustained gravity loads for heavily loaded slabs and for light frame construction supported on a podium flat plate or flat slab should be analyzed in accordance with 6.2.

6.1.2 *Slabs with beams spanning between supports on all sides*—For slabs with beams spanning between the supports on all sides, the minimum thickness h should meet the following requirements:

(a) For an average value of beam flexural stiffness α_{fm} equal to or less than 0.2, the provisions of 6.1.1 should apply.

(b) For an average value of beam flexural stiffness α_{fm} greater than 0.2 but not greater than 2.0, h should not be less than

$$h = \frac{\ell_n\left(0.8 + \frac{f_y}{200{,}000}\right)}{36 + 5\beta(\alpha_{fm} - 0.2)} \quad \text{(in.-lb)}$$

$$h = \frac{\ell_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 5\beta(\alpha_{fm} - 0.2)} \quad \text{(SI)} \tag{6.1.2a}$$

and not less than 5 in. (125 mm).

(c) For an average value of beam flexural stiffness α_{fm} greater than 2.0, h should not be less than

$$h = \frac{\ell_n\left(0.8 + \frac{f_y}{200{,}000}\right)}{36 + 9\beta} \quad \text{(in.-lb)}$$

$$h = \frac{\ell_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta} \quad \text{(SI)} \tag{6.1.2b}$$

and not less than 3.5 in. (90 mm).

(d) At discontinuous edges, provide edge beam with a stiffness ratio α_f not less than 0.80, or increase the minimum thickness required by Eq. (6.1.2a) or (6.1.2b) by at least 10 percent in the panel with a discontinuous edge.

The value of ℓ_n in (b) and (c) is the length of the clear span in the long direction measured face-to-face of the beams. The term β in (b) and (c) is the ratio of clear spans in the long-to-short direction of the slab.

For panels having a ratio of long-to-short span greater than 2, the use of Eq. (6.1.2a) and (6.1.2b), which express the minimum thickness as a fraction of the long span, may give unreasonable results. Use the rules for one-way construction for these.

6.2—Deflection analysis

Slab thickness less than the minimum required by 6.1.1 and 6.1.2 is permitted where computed deflections do not exceed the maximum permissible computed deflections limits of ACI 318-14, Section 8.3.2.1 and Table 24.2.2. When computing deflections, consider the size and shape of the panel, conditions of support, nature of restraints at the panel edges, and state of cracking.

The calculation of deflections for slabs is complicated even when linear elastic behavior can be assumed. For immediate deflections of nonprestressed slabs, the values of E_cI and I_e specified in ACI 318-14, Section 24.2.3 can be used. For prestress slabs, I_g may be used when the calculated tensile stress is less than the modulus of rupture. However, other procedures and values of the stiffness E_cI may be used if they result in predictions of deflection in reasonable agreement with the results of comprehensive tests. For lightly reinforced sections, which are common in slab positive moment regions, the I_e calculated based on ACI 318-14, 24.2.3.5 for beams overestimates the slab stiffness. For a slab with minimum reinforcement, the fully cracked section moment of inertia, I_{cr}, is approximately one-tenth the gross section moment of inertia, I_g. For typical beams, the ratio of I_{cr} is one-fourth to one-third the effective moment of inertia, I_e. Tests conducted in Canada showed that the traditional Branson's equation, ACI 318-14, Eq. (24.2.3.5a) for the calculation of I_e, underestimates the deflection of slabs (Bischoff and Scanlon 2008).

ACI 435R presents three approaches in calculating the immediate deflection: 1) classical solution; 2) simplified crossing beam analogies; and 3) finite elements. Under the crossing beam analogy, deflection of two-way slabs can be approximated by considering the column and middle strips in each of the orthogonal directions. The total static moment given in Eq. (3.1.3) is divided among column strips and middle strips using methods described under 3.2.1 or 3.2.2. Deflection can then be calculated in accordance with ACI 318-14, Sections 24.2.3.5 and 24.2.3.6.

6.2.1 *Long-term deflection*—Because available data on long-term deflections of slabs are too limited to justify more elaborate procedures, the additional long-term deflection for two-way construction can be computed using the multipliers given in ACI 318-14, Section 24.2.4.1.1. Engineers are cautioned that the long-term multipliers, which work reasonably well for beams, can be much too small for lightly reinforced sections or when the positive reinforcement ratio approaches minimum reinforcement ratio. A major factor in the deflection performance of slabs and plates as compared to beams is due to the impact of their relative stiffness effect on their cracking development. Uncracked sections subjected to initial loads produce tensile stresses approaching the modulus of rupture. Thereafter, a section is partially cracked and becomes fully cracked under long-term sustained loads with increased comparative deflection behavior in the slabs. Note also that the residual deflection of concrete slabs depends on the sequence of construction, shoring and reshoring, and proper curing techniques during construction. To preserve the integrity of the structural slab and plate, serviceability requirements through control of cracking and deflection in the applicable provisions of ACI 435R and ACI 224R should be used.

Limited data exist on long-term deflection of slabs and plates so as to justify elaborate computational procedures, including accounting for factors affecting time-dependent deflections. An alternative method is the use of ACI 318-14, Section 24.2.4.1.1. However, understand that the long-term multipliers, while working reasonably well for beams, can be too small for lightly-reinforced sections or when the positive reinforcement ratio approaches the minimum reinforcement ratio.

6.2.2 *Post-tensioned slabs*—Unlike nonprestressed concrete flat plates or flat slabs, many variations are possible in unbonded post-tensioned flat plates/slabs. Various strand placement patterns are possible, although most slabs are built with banded tendon arrangements. Various strand profiles are used, defined by the positive and negative eccentricities and points of contraflexure of the tendons. Treating discontinuous ends and strand terminations at slab openings requires careful consideration. The minimum average effective compressive stress is 125 psi (0.9 MPa) according to ACI 318-14, Section 8.6.2.1, but higher values may be required for specific cases. Although deflection is not a dominant problem, deflection calculations are required by ACI 318, Sections 24.2.3.8 and 24.2.3.9, for post-tensioned slabs. Slabs are generally thinner and spans are longer than nonprestressed concrete slabs; slab thicknesses of L/45 are common. Because thinner slabs are more flexible, they may be more susceptible to issues of floor vibration.

6.3—Crack control in reinforced two-way action structural slabs and plates

Crack control in concrete slabs at service load levels is as important as controlling deflection. It is closely connected to reducing sectional stiffness, upholding the integrity of the structure, retaining reinforcement corrosion, and preventing other detrimental effects that lead to ultimate loss of serviceability. Microcracking in concrete starts at an early load level of approximately 10 percent of the service load, which eventually leads to visible macrocracks.

Design equations for beams underestimate the crack width developed in two-way slabs and plates, and do not educate

Table 6.3—Guide to tolerable flexural crack widths for reinforced concrete under service loads*

Exposure condition	Crack width, in. (mm)
Dry air or protective membrane	0.016 (0.41)
Humidity, moist air, soil	0.012 (0.30)
Deicing chemicals	0.007 (0.18)
Seawater and seawater spray, wetting and drying	0.006 (0.15)
Water-retaining structures†	0.004 (0.10)

*Nawy (1968) and ACI 224R-01, Table 4.1.

†Excludes nonpressure pipes.

Note: Expect a portion of the cracks in the structure to exceed these values. With time, a significant portion could exceed these values. These are general guidelines for design to be used in conjunction with sound engineering judgment.

the design engineer on how to space reinforcement. Extensive research and tests to failure of more than 100 two-way plates have shown that cracks in such structural members are controlled primarily by the steel stress level and the spacing of reinforcement in the two perpendicular directions. In addition, the clear concrete cover in two-way slabs and plates is nearly constant (20 mm [3/4 in.] for most interior slabs), whereas it is a major variable in crack control equations for beams (Nawy and Blair 1971; Nawy 2011; ACI 224R).

The yield-line cracks described in 3.4 are generally wide and almost fully developed at approximately 30 percent of the service load level. To delay their development until the load reaches the expected nominal flexural strength, design closely-spaced reinforcement in the two orthogonal directions so the crack pattern in the two-way slab or plate at service and low overload levels is an image of the orthogonal reinforcement. The crack widths in such designs will, therefore, be controlled within the tolerable limits for various environmental conditions, as shown in Table 6.3.

When selecting reinforcement spacing for a two-way floor system, specify flexural reinforcement in the N-S and E-W directions for the same volume of steel using smaller-diameter bars at closer spacing. It is significantly more effective in reducing the crack width to an accepted level through the proper selection of the reinforcement grid and by placing closest to the tensile face of the slab. The crack width expression in terms of grid spacing, reinforcement percentage ratio, bar diameter, and magnitude of concrete cover as presented in Section 4.3 of ACI 224R (Eq. (4-15) and (4-16)) should facilitate the choice of proper proportioning of the structural slab with the choice of the appropriate size and spacing of the bars in the orthogonal directions for effective crack control. A 12 in. (300 mm) maximum spacing of the steel bar reinforcement grid is recommended to prevent the formation of detrimental wide yield-line cracks until the ultimate load is reached.

CHAPTER 7—DESIGN OF SLAB-COLUMN FRAMES UNDER LATERAL FORCES

7.1—General

This section addresses two-way slab systems without beams only, or flat plates, and recommends analysis, design, and reinforcement detailing methods for slab-column frames under combined effects of lateral and gravity loads. Slab-column frames could be used as part of an ordinary or intermediate seismic-force-resisting system, although they are usually only used to resist wind loads in combination with gravity loads. A slab-column structure acting as the seismic-force-resisting system is likely to be far too flexible for higher seismic design categories (SDCs) and would not deliver performance consistent with other requirements.

For earthquake ground motions, slab-column framing systems designed according to Chapters 1 through 6 can be used in ordinary moment frames, and are appropriate for SDC A or B. If a slab-column frame meets additional requirements described in this chapter, it may be considered as an intermediate moment frame and is allowed for structures assigned to SDC C (ASCE 7-10, Section 12.2). Additional requirements are related to the distribution of slab moments, arrangement of slab reinforcement, and punching-shear-related issues.

For structures assigned to SDC D, E, or F, slab-column frames without beams are generally not permitted as part of seismic-force-resisting systems (ACI 318-14, Section R18.2), with limited exceptions under ASCE 7-10, Section 12.2. For example, one exception would be intermediate moment frames in a dual system with special reinforced concrete shear walls less than 100 ft (30.5 m) in height. The reasoning behind this is the slab-column frames cannot be detailed for the level of energy dissipation and ductility demanded for special moment frames. In the case of ductility demand, this chapter describes punching-shear-related recommendations for slab-column connections, along with the design story drift estimated for the seismic-force-resisting system.

Under wind loads, all structural members are typically expected to behave essentially within their elastic range of response. Therefore, while the design forces induced by wind in slab-column frames may be determined based on similar analysis procedures that are used for the earthquake-resistant design (7.2), other design and detailing recommendations intended for ductility and redundancy of frames under earthquake ground motions do not apply for wind design (7.3 through 7.5).

7.2—Analysis of slab-column frames under lateral forces

For lateral forces, analysis of frames should consider effects of cracking and reinforcement on stiffness of frame members (7.2.2).

7.2.1 *Effective slab width model*—An equivalent slab-beam is a flexural member having a rectangular section with its width and depth dimensions equal to the effective slab width and slab thickness. Previous research (Vanderbilt and Corley 1983; Grossman 1997; Hwang and Moehle 2000; Dovich and Wight 2005) proposed various methods for determining the effective slab width in consideration of slab-column connection geometry and expected drift ratio under serviceability and ultimate limit states.

In the modified equation for b_e by Grossman (1997), clear span lengths ℓ_n are used in place of center-to-center span lengths ℓ_1, with the assumption that slab-column joints are rigid. The effective slab width can be expressed as

$$b_e = \left[0.3\ell_n + c_1 x + \frac{(c_2 - c_1)}{2}\right]\left(\frac{d}{0.9h}\right)K_{FP} \qquad (7.2.1)$$

where χ—the ratio of span ℓ_2/ℓ_1—should not be taken greater than 1.0, and K_{FP}, which is modification factor accounting for joint confinement, is equal to 0.8 and 0.6 for edge and corner connections, respectively. Here, ℓ_1 is taken as the average of the lengths of the two spans in front and back of the column, and ℓ_2 is the average of the lengths of the two transverse spans at the sides of the column, where both ℓ_1 and ℓ_2 are measured center-to-center of supports parallel and perpendicular to lateral loading, respectively. For exterior or corner connections with the slab edge parallel to the direction of lateral loading, the effective slab width calculated by Eq. (7.2.1) is adjusted by multiplying by $(\ell_3 + \ell_2/2)/\ell_2$, where ℓ_3 is the distance from column centerline to edge of slab. The width of an equivalent slab-beam supported by two adjacent columns is then taken equal to the average of the two values determined by Eq. (7.2.1) at the supports. More detailed model descriptions are provided in the paper by Grossman (1997). Different stiffness degradation levels of flat plates at various drifts were also proposed based on the tests by Hwang and Moehle (2000).

Equation (7.2.1) is intended for slab-column frames subjected to service wind or earthquake-induced forces, reduced by the response reduction factor R, which are expected to cause drift levels of approximately 0.25 percent. Therefore, using this model for service level analysis is considered appropriate. For strength-based analysis, flexural stiffness of equivalent slab-beams was considered to be reduced by 30 percent on average due to more cracking in the slab (Grossman 1997). Also, the effective slab width estimated using Eq. (7.2.1) should be modified when slab openings exist near supports.

7.2.2 *Slab stiffness reduction*—The analysis of slab-column frames under lateral forces should consider effects of cracking and reinforcement on the stiffness of frame members. Cracking reduces the stiffness of slab members and increases lateral flexibility when lateral forces act on the structure. The selection of appropriate effective stiffness values for reinforced concrete frame members has dual purposes to: 1) provide realistic estimates of lateral deflections; and 2) determine distribution of forces and moments on the frame members. A detailed nonlinear analysis of the structure would adequately capture these two effects. An approximate method to estimate an equivalent nonlinear lateral deflection using linear analysis is to reduce the modeled stiffness of the concrete members in the structure. One reasonable option that considers the reduced stiffness of the elements is to calculate the secant stiffness value to the point of yielding of reinforcement for the member, or the secant value to a point before yielding of the reinforcement if analysis demonstrates yielding is not expected for the given loading condition. When the analysis is used to determine design drifts or moment magnifications caused by wind or earthquake-induced forces, lower-bound slab stiffnesses should be assumed. When the analysis is used to study interactions of slabs with other framing elements, such as structural walls, it is appropriate to consider a range of slab stiffnesses so that the relative importance of slabs on those interactions can be assessed. For nonprestressed slabs, it is normally appropriate to reduce slab bending stiffness to between one-half and one-fourth of uncracked stiffness values based on gross section properties or based on limits prescribed in ACI 318-14, Section 6.6.3.1.1. For prestressed construction, stiffnesses greater than those of cracked, nonprestressed slabs may be appropriate.

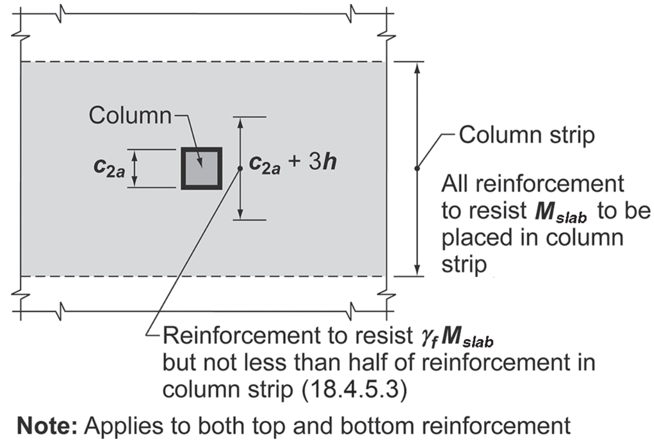

Fig. 7.2.1—Location of reinforcement in slabs.

7.3—Arrangement of reinforcement in slabs for intermediate moment frames

This section details the arrangement of slab reinforcement for two-way slab-column framing systems subjected to earthquake-induced forces and used as intermediate moment frames. Application of the provisions for two-way slabs without beam is illustrated in Fig. 7.2.1, 7.3a, and 7.3b.

7.3.1 *Slab reinforcement at support*—The factored slab moments at supports, including earthquake effects, E, are determined for load combinations $1.2D + 1.0E + 1.0L + 0.2S$ and $0.9D + 1.0E$. Using these load combinations may result in moments requiring top and bottom reinforcement at the supports. Reinforcement provided to resist M_{sc} should be placed within the column strip. The moment M_{sc} refers (for a given design load combination with E acting in one horizontal direction) to that portion of the factored slab moment that is balanced by the supporting members at a joint. It is not necessarily equal to the total design moment at a support for a load combination including earthquake effect.

As described in 4.7, only a fraction of the moment M_{sc} is assigned to the slab effective width. Reinforcement placed within the effective width should resist $\gamma_f M_{sc}$. The effective slab width for exterior and corner connections should not extend beyond the column face a distance greater than c_t measured perpendicular to the slab span. For edge and

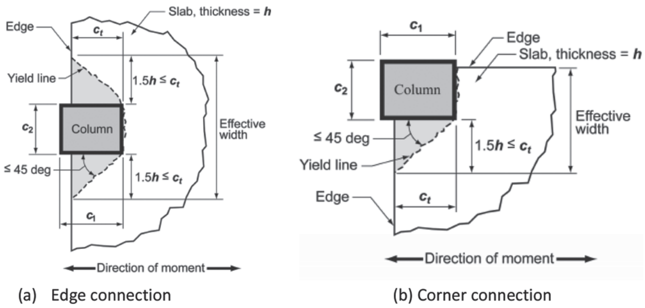

Fig. 7.3a—Effective width for reinforcement placement in edge and corner connections.

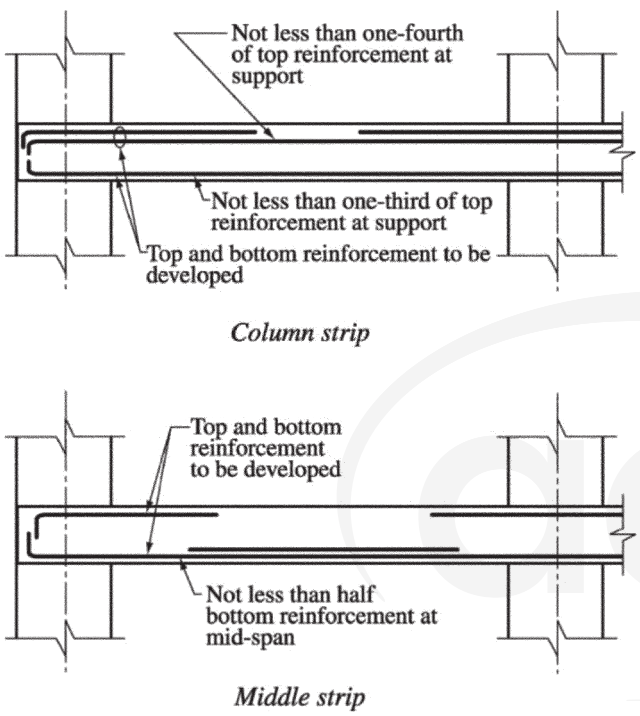

Fig. 7.3b—Arrangement of reinforcement in slabs.

corner connections, flexural reinforcement perpendicular to the edge is not considered fully effective unless it is placed within the effective slab width. At least one-half of the reinforcement in the column strip at the support should be placed within the effective slab width given in 4.7.

7.3.2 *Reinforcement continuity*—At least one-fourth of the top reinforcement at the support in the column strip should be continuous throughout the span. Also, continuous bottom reinforcement in the column strip should be at least one-third of the top reinforcement at the support in the column strip. At least one-half of all bottom middle strip reinforcement and all bottom column strip reinforcement at midspan needs to be continuous and should develop f_y at face of support. At discontinuous edges of the slab, all top and bottom reinforcement at support should be developed at the face of the support (Fig. 7.3b).

7.3.3 *Limit for factored gravity shear at support*—At the critical sections for columns defined in 4.5, two-way shear caused by factored gravity loads should not exceed $0.4\phi V_c$, where V_c should be calculated as defined in 4.8 for nonprestressed slabs and in 4.9 for prestressed slabs. This can be waived if the slab design follows 7.4.

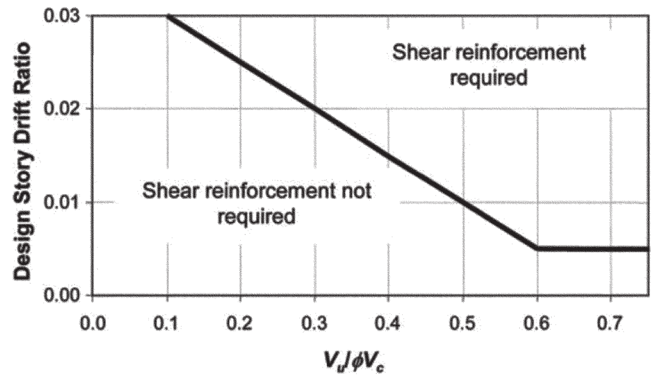

Fig. 7.4—Illustration of slab-column criterion.

7.4—Slab-column frames not designated as part of the seismic-force-resisting system

This section applies only to structures assigned to seismic design category (SDC) D, E, or F. According to ASCE 7-10, Section 12.12.5, all structural members not designated as a part of the seismic-force-resisting system should be designed to support gravity loads while subjected to design displacement. The principle behind this provision is to allow flexural yielding of slabs under design displacement, and to provide sufficient shear strength in slabs that yield so that the slabs continue to support gravity loads.

For slab-column connections of two-way slabs without beams in structures assigned to SDC D, E, or F, slab shear reinforcement (5.8.1 and 5.8.2), providing V_s not less than $3.5\sqrt{f_c'}\,b_o d$ (psi) $(0.29\sqrt{f_c'}\,b_o d$ [MPa]), should extend at least four times the slab thickness from the face of the support, unless:

a) The information in 4.7 using the design shear V_{ug} and the induced moment transferred between the slab and column under the design displacement is used

b) The design story drift ratio does not exceed the larger of 0.005 and $(0.035 - 0.05(V_{ug}/\phi V_c))$ (Fig. 7.4).

The design story drift ratio should be taken as the larger of the design story drift ratios of the adjacent stories above and below the slab-column connection. The value of V_{ug} is calculated for the load combination $1.2D + 1.0L + 0.2S$. The load factor on the live load, L, needs to be reduced to 0.5 except for garages, areas occupied as places of public assembly, and all areas where L is greater than 100 lb/ft^2 (5.0 kPa).

The induced moment is calculated to occur at the slab-column connection when subjected to design displacement. Effects of cracking and reinforcement on stiffness of frame members should be considered in analysis of slab-column frames in resisting seismic forces.

Condition b) does not require calculation of induced moments, and is based on research that identifies the likelihood of punching shear failure considering the story drift ratio and shear due to gravity loads. Figure 7.4 illustrates condition b). This can be accomplished by adding slab shear reinforcement, increasing slab thickness, changing the design to reduce the design story drift ratio, or a combination of these. If column capitals, drop panels, shear caps, or other changes in slab thickness are used, the guidelines in this section need to be evaluated at all potential critical sections, as described in 4.4.

The optional design permitted by ACI 318-14, Section 18.14.5.1 is concerned with the ductility (design story drift); it does not calculate shear forces or unbalanced moments associated with design story drift and, therefore, can permit connections without verifying that they possess the strength required in other sections of the Code. For this reason, the design option for condition a) is recommended because it verifies adequate strength. It is suggested that minimum shear reinforcement should be specified, including the amount of reinforcement and the zone where it should be placed, to ensure adequate ductility is provided.

7.5—Transfer of moments to column

Brittle punching failure can occur due to the transfer of shear forces combined with unbalanced moments between slabs and columns. During an earthquake, significant horizontal displacement of a flat plate-column connection may occur, resulting in unbalanced moments that induce additional slab shear. The displacement-induced unbalanced moments and resulting shear forces at flat plate-column connections should be considered in design to prevent brittle punching shear failure. Even when an independent lateral-force-resisting system is provided, flat plate-column connections should be designed to accommodate the moments and shear forces associated with the displacements during earthquakes (ACI 421.2R). Consideration of detailing ensures ductile behavior of building system.

CHAPTER 8—REFERENCES

ACI Committee documents and documents published by other organizations are listed first by document number, full title, and year of publication followed by authored documents listed alphabetically.

American Concrete Institute

224R-01(08)—Control of Cracking in Concrete Structures

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318-14—Building Code Requirements for Structural Concrete and Commentary

318-77—Building Code Requirements for Reinforced Concrete and Commentary

318-95—Building Code Requirements for Structural Concrete and Commentary

421.1R-08—Guide to Shear Reinforcement for Slabs

421.2R-10—Guide to Seismic Design of Punching Shear Reinforcement in Flat Plates

423.3R-05—Recommendations for Concrete Members Prestressed with Unbonded Tendons

435R-95(00)—Control of Deflection in Concrete Structures (Appendix B added 2003)

American Society of Civil Engineers

ASCE 7-10—Minimum Design Loads for Buildings and Other Structures

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APPENDIX A—EXCERPT BUILDING CODE PROVISIONS

The following text are excerpts from ACI 318-14, included in this guide to enhance the use of this guide as a source document in design of two- way slabs. Future revisions of this guide will merge the current commentary into the guide.

A.1—Direct design method (ACI 318-14, Section 8.10)

8.10—Direct design method

8.10.1 *General*

8.10.1.1 Two-way slabs satisfying the limits in 8.10.2 shall be permitted to be designed in accordance with this section.

8.10.1.2 Variations from the limitations in 8.10.2 shall be permitted if demonstrated by analysis that equilibrium and geometric compatibility are satisfied, the design strength at every section is at least equal to the required strength, and serviceability conditions, including limits on deflection, are met.

8.10.1.3 Circular or regular polygon-shaped supports shall be treated as square supports with the same area.

8.10.2 *Limitations for use of direct design method*

8.10.2.1 There shall be at least three continuous spans in each direction.

8.10.2.2 Successive span lengths measured center-to-center of supports in each direction shall not differ by more than one-third the longer span.

8.10.2.3 Panels shall be rectangular, with the ratio of longer to shorter panel dimensions, measured center-to-center of supports, not to exceed 2.

8.10.2.4 Column offset shall not exceed 10 percent of the span in direction of offset from either axis between centerlines of successive columns.

8.10.2.5 All loads shall be due to gravity only and uniformly distributed over an entire panel.

8.10.2.6 Unfactored live load shall not exceed two times the unfactored dead load.

8.10.2.7 For a panel with beams between supports on all sides, Eq. (8.10.2.7a) shall be satisfied for beams in the two perpendicular directions.

$$0.2 \le \frac{\alpha_{f1}\ell_2^2}{\alpha_{f2}\ell_1^2} \le 5.0 \qquad (8.10.2.7a)$$

where $\boldsymbol{\alpha}_{f1}$ and $\boldsymbol{\alpha}_{f2}$ are calculated by:

$$\alpha_f = \frac{E_{cb}I_b}{E_{cs}I_s} \qquad (8.10.2.7b)$$

8.10.3 *Total factored static moment for a span*

8.10.3.1 Total factored static moment $\boldsymbol{M_o}$ for a span shall be calculated for a strip bounded laterally by the panel centerline on each side of the centerline of supports.

8.10.3.2 The absolute sum of positive and average negative $\boldsymbol{M_u}$ in each direction shall be at least:

Table 8.10.4.2—Distribution coefficients for end spans

	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports		Exterior edge fully restrained
			Without edge beam	With edge beam	
Interior negative	0.75	0.70	0.70	0.70	0.65
Positive	0.63	0.57	0.52	0.50	0.35
Exterior negative	0	0.16	0.26	0.30	0.65

$$M_o = \frac{q_u \ell_2 \ell_n^2}{8} \quad (8.10.3.2)$$

8.10.3.2.1 In Eq. (8.10.3.2), ℓ_n is the clear span length in the direction that moments are considered, shall extend from face to face of columns, capitals, brackets, or walls, and shall be at least $0.65\ell_1$.

8.10.3.2.2 In Eq. (8.10.3.2), if the transverse span of panels on either side of the centerline of supports varies, ℓ_2 shall be taken as the average of adjacent transverse spans.

8.10.3.2.3 In Eq. (8.10.3.2), if the span adjacent and parallel to a slab edge is being considered, the distance from edge to panel centerline shall be substituted for ℓ_2.

8.10.4 *Distribution of total factored static moment*

8.10.4.1 In an interior span, M_o shall be distributed as follows: $0.65M_o$ to negative moment and $0.35M_o$ to positive moment.

8.10.4.2 In an end span, M_o shall be distributed in accordance with Table 8.10.4.2.

8.10.4.3 Modification of negative and positive factored moments by up to 10 percent shall be permitted if the total factored static moment for a panel, M_o, in the direction considered is at least that calculated by Eq. (8.10.3.2). Moment redistribution in accordance with 6.6.5 is not permitted.

8.10.4.4 Critical section for negative M_u shall be at the face of rectangular supports.

8.10.4.5 Negative M_u shall be the greater of the two interior negative M_u calculated for spans framing into a common support unless an analysis is made to distribute the unbalanced moment in accordance with stiffnesses of adjoining elements.

8.10.4.6 Edge beams or edges of slabs shall be designed to resist in torsion their share of exterior negative M_u.

8.10.5 *Factored moments in column strips*

8.10.5.1 The column strip shall resist the portion of interior negative M_u in accordance with Table 8.10.5.1.

8.10.5.2 The column strip shall resist the portion of exterior negative M_u in accordance with Table 8.10.5.2.

$$\beta_t = \frac{E_{cb} C}{2E_{cs} I_s} \quad (8.10.5.2a)$$

Table 8.10.5.1—Portion of interior negative M_u in column strip

	ℓ_2/ℓ_1		
$\alpha_{f1}\ell_2/\ell_1$	0.5	1.0	2.0
0	0.75	0.75	0.75
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

Table 8.10.5.2—Portion of exterior negative M_u in column strip

		ℓ_2/ℓ_1		
$\alpha_{f1}\ell_2/\ell_1$	β_t	0.5	1.0	2.0
0	0	1.0	1.0	1.0
	≥2.5	0.75	0.75	0.75
≥1.0	0	1.0	1.0	1.0
	≥2.5	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown. β_t is calculated using Eq. (8.10.5.2a), where C is calculated using Eq. (8.10.5.2b).

Table 8.10.5.5—Portion of positive M_u in column strip

	ℓ_2/ℓ_1		
$\alpha_{f1}\ell_2/\ell_1$	0.5	1.0	2.0
0	0.60	0.60	0.60
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

$$C = \sum\left(1 - 0.63\frac{x}{y}\right)\frac{x^3 y}{3} \quad (8.10.5.2b)$$

8.10.5.3 For T- or L-sections, it shall be permitted to calculate the constant C in Eq. (8.10.5.2b) by dividing the section, as given in 8.4.1.8, into separate rectangular parts and summing the values of C for each part.

8.10.5.4 If the width of the column or wall is at least $(3/4)\ell_2$, negative M_u shall be uniformly distributed across ℓ_2.

8.10.5.5 The column strip shall resist the portion of positive M_u in accordance with Table 8.10.5.5.

8.10.5.6 For slabs with beams between supports, the slab portion of column strips shall resist column strip moments not resisted by beams.

8.10.5.7 *Factored moments in beams*

8.10.5.7.1 Beams between supports shall resist the portion of column strip M_u in accordance with Table 8.10.5.7.1.

8.10.5.7.2 In addition to moments calculated according to 8.10.5.7.1, beams shall resist moments caused by factored loads applied directly to the beams, including the weight of the beam stem above and below the slab.

8.10.6 *Factored moments in middle strips*

8.10.6.1 That portion of negative and positive factored moments not resisted by column strips shall be proportionately assigned to corresponding half middle strips.

8.10.6.2 Each middle strip shall resist the sum of the moments assigned to its two half middle strips.

8.10.6.3 A middle strip adjacent and parallel to a wall-supported edge shall resist twice the moment assigned to the half middle strip corresponding to the first row of interior supports.

Table 8.10.5.7.1—Portion of column strip M_u in beams

$\alpha_{f1}\ell_2/\ell_1$	Distribution coefficient
0	0
≥1.0	0.85

Note: Linear interpolation shall be made between values shown.

8.10.7 *Factored moments in columns and walls*

8.10.7.1 Columns and walls built integrally with a slab system shall resist moments caused by factored loads on the slab system.

8.10.7.2 At an interior support, columns or walls above and below the slab shall resist the factored moment calculated by Eq. (8.10.7.2) in direct proportion to their stiffnesses unless a general analysis is made.

$$M_{sc} = 0.07[(q_{Du} + 0.5q_{Lu})\ell_2\ell_n^2 - q_{Du}'\ell_2'(\ell_n')^2] \quad (8.10.7.2)$$

where q_{Du}', ℓ_2', and ℓ_n' refer to the shorter span.

8.10.7.3 The gravity load moment to be transferred between slab and edge column in accordance with 8.4.2.3 shall not be less than $0.3M_o$.

8.10.8 *Factored shear in slab systems with beams*

8.10.8.1 Beams between supports shall resist the portion of shear in accordance with Table 8.10.8.1 caused by factored loads on tributary areas in accordance with Fig. 8.10.8.1.

8.10.8.2 In addition to shears calculated according to 8.10.8.1, beams shall resist shears caused by factored loads applied directly to the beams, including the weight of the beam stem above and below the slab.

8.10.8.3 Calculation of required slab shear strength based on the assumption that load is distributed to supporting beams in accordance with 8.10.8.1 shall be permitted. Shear resistance to total V_u occurring on a panel shall be provided.

A.2—Equivalent frame method (ACI 318-14, Section 8.11)

8.11—Equivalent frame method

8.11.1 *General*

8.11.1.1 All sections of slabs and supporting members in two-way slab systems designed by the equivalent frame method shall resist moments and shears obtained from an analysis in accordance with 8.11.2 through 8.11.6.

8.11.1.2 Live load shall be arranged in accordance with 6.4.3.

8.11.1.3 It shall be permitted to account for the contribution of metal column capitals to stiffness, resistance to moment, and resistance to shear.

8.11.1.4 It shall be permitted to neglect the change in length of columns and slabs due to direct stress, and deflections due to shear.

8.11.2 *Equivalent frames*

8.11.2.1 The structure shall be modeled by equivalent frames on column lines taken longitudinally and transversely through the building.

8.11.2.2 Each equivalent frame shall consist of a row of columns or supports and slab-beam strips bounded laterally by the panel centerline on each side of the centerline of columns or supports.

Table 8.10.8.1—Portion of shear resisted by beam

$\alpha_{f1}\ell_2/\ell_1$	Distribution coefficient
0	0
≥1.0	1.0

Note: Linear interpolation shall be made between values shown.

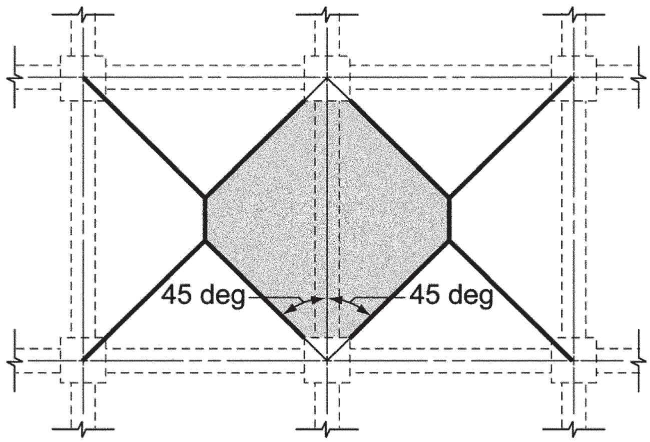

Fig. 8.10.8.1—Tributary area for shear on an interior beam.

8.11.2.3 Frames adjacent and parallel to an edge shall be bounded by that edge and the centerline of the adjacent panel.

8.11.2.4 Columns or supports shall be assumed to be attached to slab-beam strips by torsional members transverse to the direction of the span for which moments are being calculated and extending to the panel centerlines on each side of a column.

8.11.2.5 Analysis of each equivalent frame in its entirety shall be permitted. Alternatively, for gravity loading, a separate analysis of each floor or roof with the far ends of columns considered fixed is permitted.

8.11.2.6 If slab-beams are analyzed separately, it shall be permitted to calculate the moment at a given support by assuming that the slab-beam is fixed at supports two or more panels away, provided the slab continues beyond the assumed fixed supports.

8.11.3 *Slab-beams*

8.11.3.1 The moment of inertia of slab-beams from the center of the column to the face of the column, bracket, or capital shall be assumed equal to the moment of inertia of the slab-beam at the face of the column, bracket, or capital divided by the quantity $(1 - c_2/\ell_2)^2$, where c_2 and ℓ_2 are measured transverse to the direction of the span for which moments are being determined.

8.11.3.2 Variation in moment of inertia along the axis of slab-beams shall be taken into account.

8.11.3.3 It shall be permitted to use the gross cross-sectional area of concrete to determine the moment of inertia of slab-beams at any cross section outside of joints or column capitals.

8.11.4 *Columns*

8.11.4.1 The moment of inertia of columns from top to bottom of the slab-beam at a joint shall be assumed to be infinite.

8.11.4.2 Variation in moment of inertia along the axis of columns shall be taken into account.

8.11.4.3 It shall be permitted to use the gross cross-sectional area of concrete to determine the moment of inertia of columns at any cross section outside of joints or column capitals.

8.11.5 *Torsional members*

8.11.5.1 Torsional members shall be assumed to have a constant cross section throughout their length consisting of the greatest of (a) through (c):

(a) A portion of slab having a width equal to that of the column, bracket, or capital in the direction of the span for which moments are being determined.

(b) For monolithic or fully composite construction, the portion of slab specified in (a) plus that part of the transverse beam above and below the slab.

(c) The transverse beam in accordance with 8.4.1.8.

8.11.5.2 Where beams frame into columns in the direction of the span for which moments are being calculated, the torsional stiffness shall be multiplied by the ratio of the moment of inertia of the slab with such a beam to the moment of inertia of the slab without such a beam.

8.11.6 *Factored moments*

8.11.6.1 At interior supports, the critical section for negative M_u in both column and middle strips shall be taken at the face of rectilinear supports, but not farther away than $0.175\ell_1$ from the center of a column.

8.11.6.2 At exterior supports without brackets or capitals, the critical section for negative M_u in the span perpendicular to an edge shall be taken at the face of the supporting element.

8.11.6.3 At exterior supports with brackets or capitals, the critical section for negative M_u in the span perpendicular to an edge shall be taken at a distance from the face of the supporting element not exceeding one-half the projection of the bracket or capital beyond the face of the supporting element.

8.11.6.4 Circular or regular polygon-shaped supports shall be assumed to be square supports with the same area for location of critical section for negative design moment.

8.11.6.5 Where slab systems within limitations of 8.10.2 are analyzed by the equivalent frame method, it shall be permitted to reduce the calculated moments in such proportion that the absolute sum of the positive and average negative design moments need not exceed the value obtained from Eq. (8.10.3.2).

8.11.6.6 It shall be permitted to distribute moments at critical sections to column strips, beams, and middle strips in accordance with the direct design method in 8.10, provided that Eq. (8.10.2.7a) is satisfied.

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